

MIRACOSTA COLLEGE

Student: _____

Notetaker: Krystal

Class: Math 150

Day/Date: May 8, 2009

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} \quad \begin{matrix} \text{(run)} \\ \text{(rise)} \end{matrix}$$

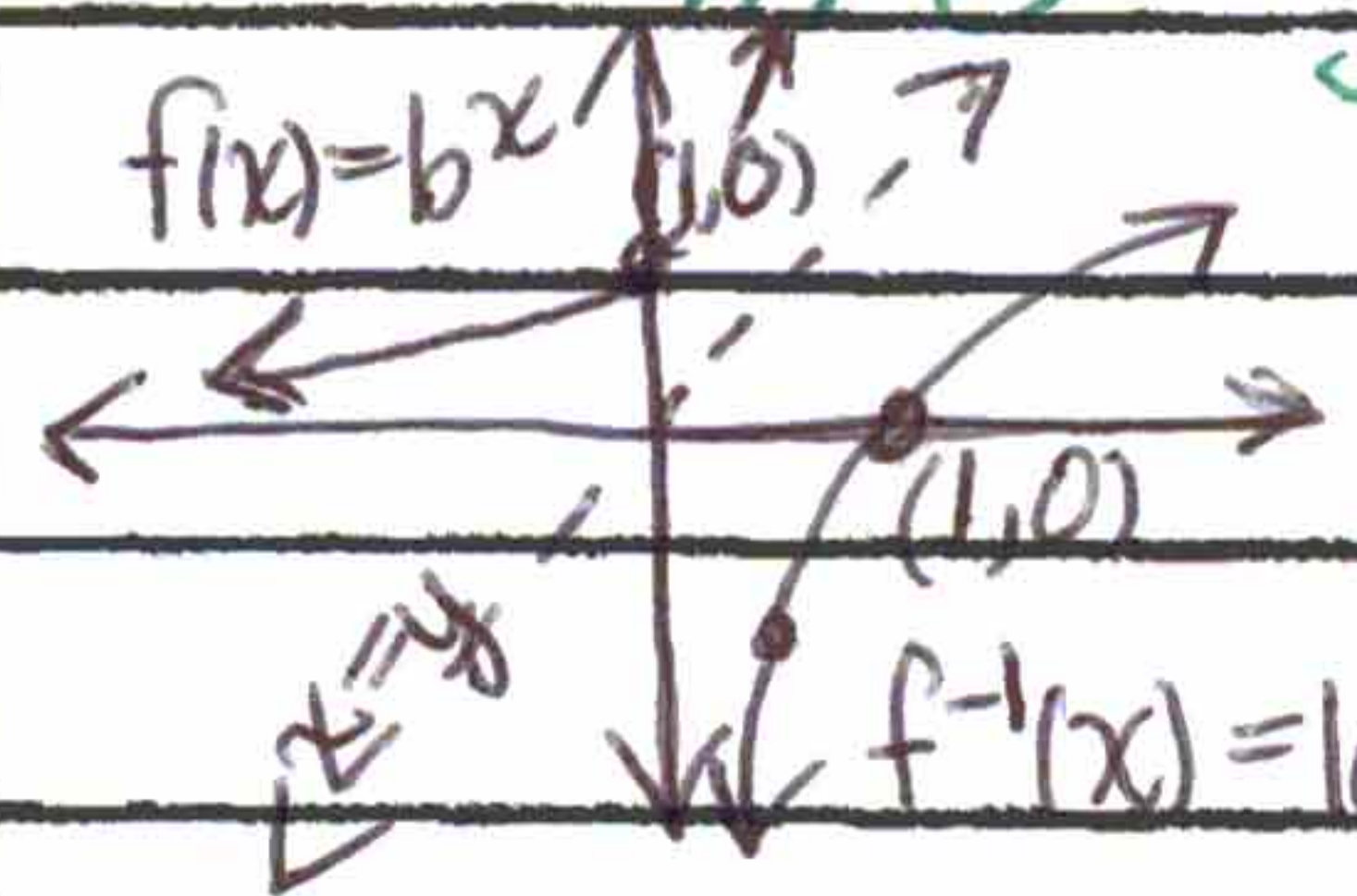
Chapter 5

- (1) $\log_b(1) = 0$
- (2) $\log_b(AB) = \log_b A + \log_b B$
- (3) $\log_b(A^n) = n \log_b A$
- (4) $\log_b(A/B) = \log_b A - \log_b B$
- (5) $\log_B(A) = \frac{\log_x A}{\log_x B} \quad x > 0, x \neq 1$

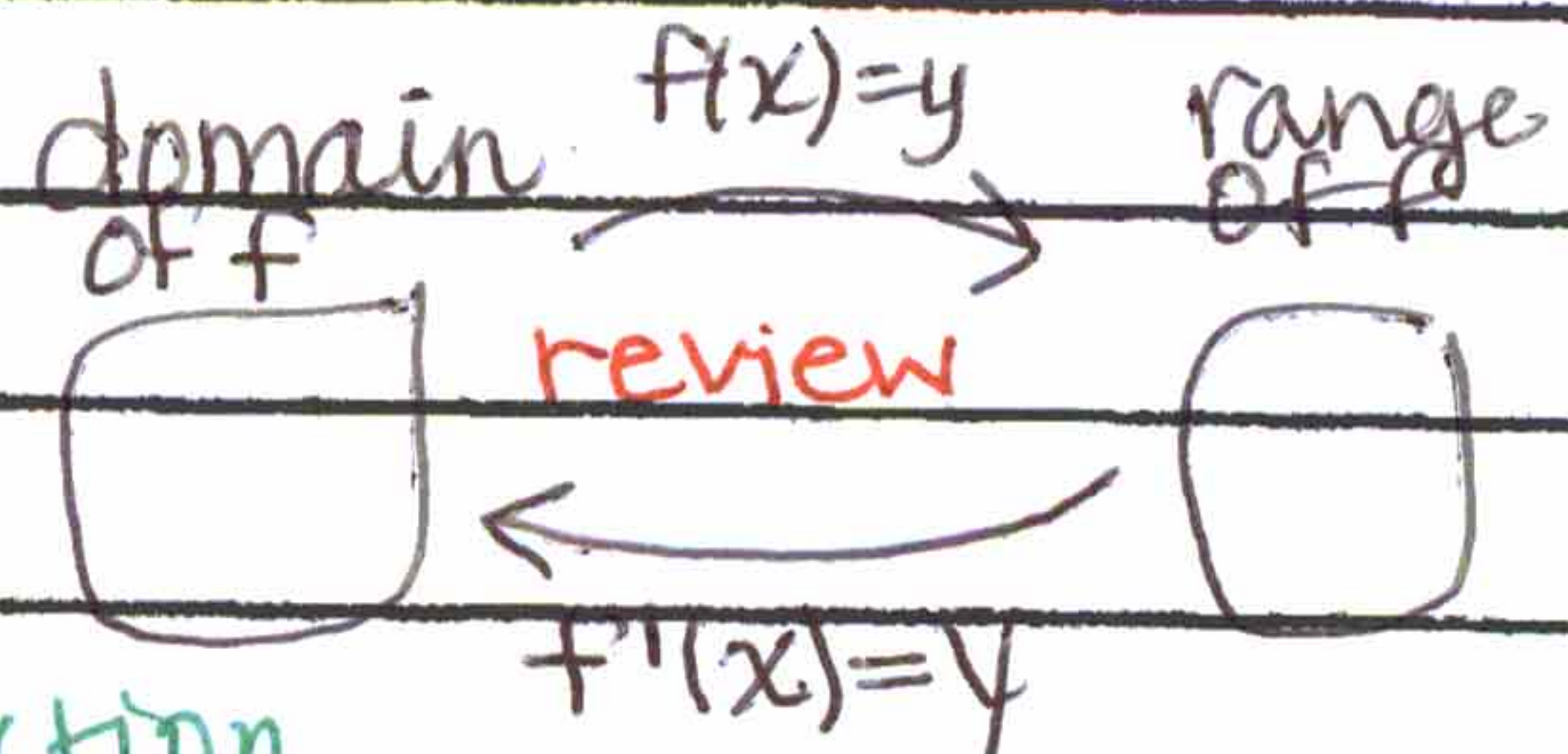
Definition of e

The letter e denotes the positive real # such that

$$\ln e = \int_1^e \frac{1}{t} dt = 1$$



all x's & y's trade position



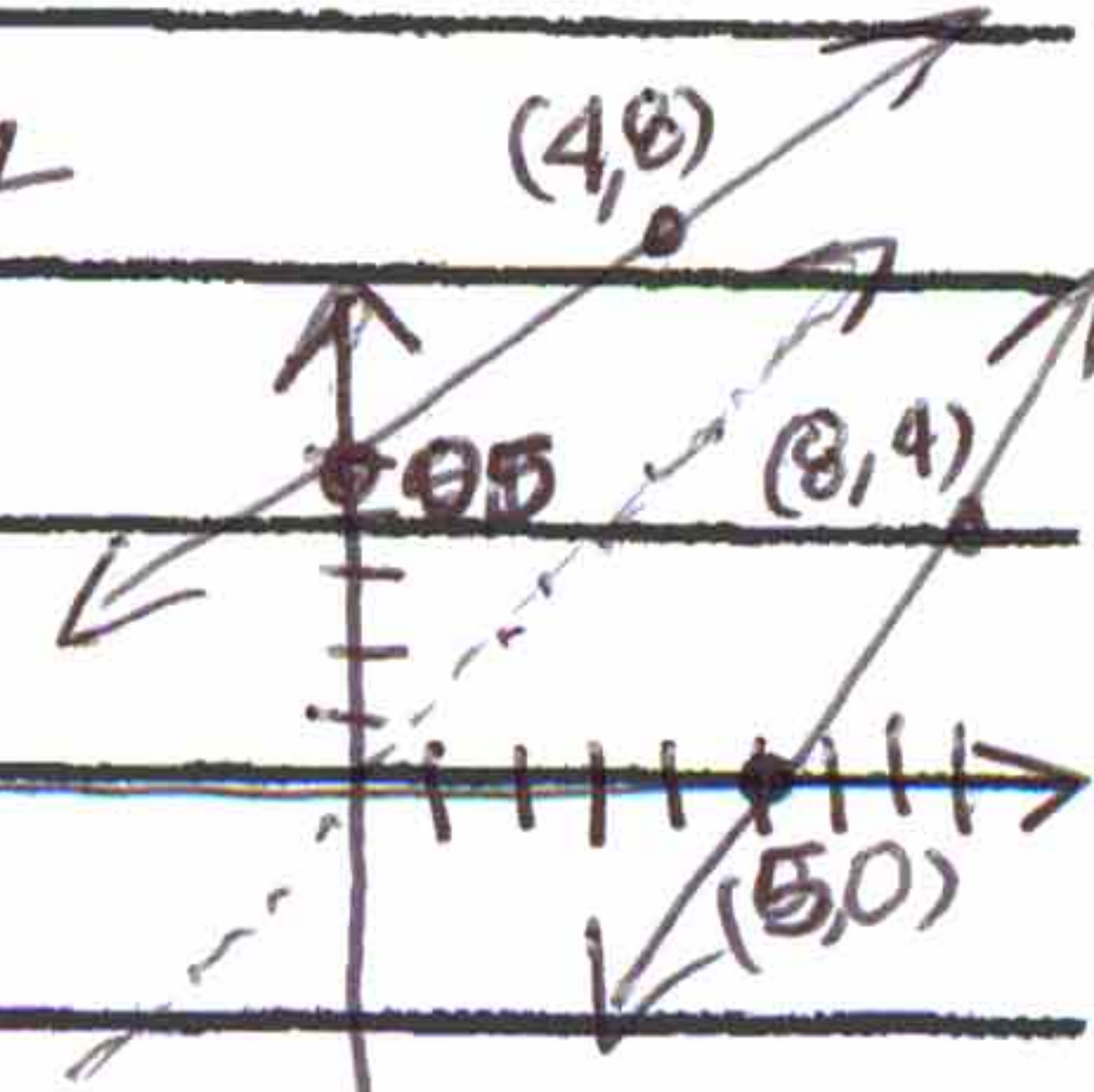
$$f(f^{-1}(x)) = x$$

$$f^{-1}(f(x)) = x$$

5.3 Existence of an Inverse function

- A.
- B.

f must pass horizontal line test
f must be a one-to-one



Guidelines for finding an Inverse f^-1

$$f(x) = \frac{3}{4}x + 5$$

$$\textcircled{1} y = \frac{3}{4}x + 5$$

$$\textcircled{2} x = \frac{3}{4}y + 5$$

$\textcircled{3}$ solve for y

$$x - 5 = \frac{3}{4}y + 5 - 5$$

$$\frac{4}{3}(x - 5) = \frac{3}{4}y \cdot \frac{4}{3}$$

$$\frac{4x - 20}{3} = y$$

$$f^{-1}(x) = \frac{4x}{3} - \frac{20}{3}$$

↑ concept is on final ↑

(5) calc. p2

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Graph inverse fn on @ calc.

y =

[2nd] → [DRAW] → [8: DRAW INV] [Y1]

$$\frac{d}{dx}(3/4^x) = 3/4^x \ln 3/4$$

$$\frac{d}{dx}(2^x) = 2^x \ln 2$$

Integrals of the Six Basic Trig fns (p.337) - familiarize

Derivative of an Inverse fn

tip: let $g(x) = f^{-1}(x)$ ↳ difficult to solve for -

do not apply differential to power rule. use substitution

$$\frac{d}{dx}[f^{-1}(x)] = \frac{d}{dx}[g(x)]$$

Important: ~~let~~ $y = f^{-1}(x)$

then $f(y) = f(f^{-1}(x))$

let $f(x) = b^x$ $b > 0$
 $b \neq 1$
 $f'(x)$

Chain $\frac{d}{dx}[f(y)] = \frac{d}{dx} x$

$$f'(y) \cdot \frac{dy}{dx} = \frac{dx}{dx}$$

$$f'(y) \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{f'(y)}$$

$$\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'(y)}$$

memorize

$$f(f^{-1}(x)) = x$$

$$f^{-1}(f(x)) = x$$

$$\frac{d}{dx}[f(x)] = \frac{d}{dx}(b^x)$$

let $y = b^x$

$$\ln(y) = \ln(b^x)$$

$$\ln(y) = x \ln(b)$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} (x \ln b)$$

$$1/y \cdot dy/dx = \ln b$$

$$dy/dx = y \ln(b) = b^x \ln b$$

* derivative of a function is its fn

$$\frac{d}{dx} e^x = e^x$$

$$\int e^x dx = e^x + C$$

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$$\frac{d}{dx} [\cos^{-1}(x)]$$

← don't like f^{-1} notation

Let $y = \cos^{-1}(x)$

$$\cos(y) = \cos(\cos^{-1}(x))$$

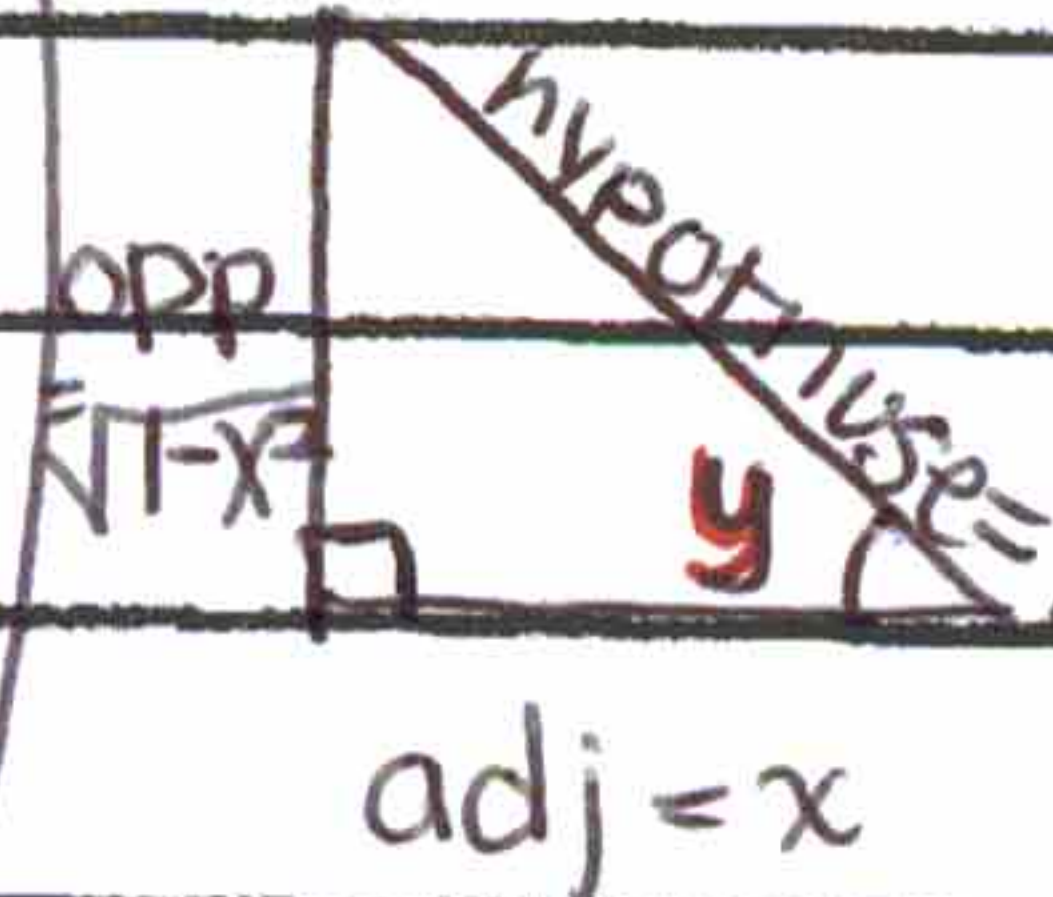
$$\cos(y) = x$$

$$\frac{dy}{dx} = \frac{1}{-\sin(\cos^{-1}(x))}$$

Implicit differentiation & chain rule

$$\frac{d}{dx} (\cos(y)) = \frac{d}{dx} (x)$$

$$-\sin(y) \cdot \frac{dy}{dx} = 1$$



$\cos y = \frac{x}{1} = \frac{\text{adj}}{\text{hyp}}$
Pythagorean theorem.

$$\frac{dy}{dx} = \frac{-1}{\sin(y)}$$

$$\frac{d}{dx} [\cos^{-1}(x)] = \frac{1}{-\sqrt{1-x^2}}$$

S.t.o memorize

Definition of natural exponential fn

Definition of Inverse Trigonometric fns (pg 371)

- the graphs p. 372

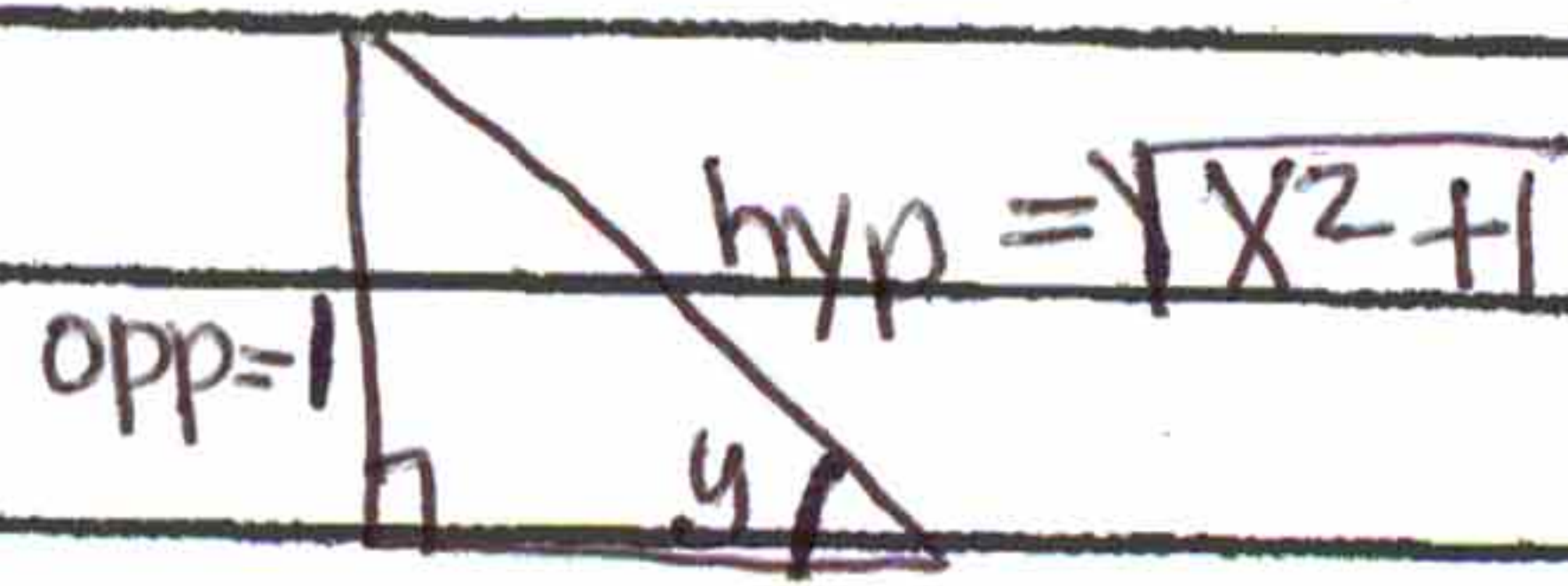
Derivatives of Inverse Trig functions (pg 374)

$$\frac{d}{dx} (\cot^{-1}(x)) =$$

Let $y = \cot^{-1}(x)$

$$\cot(y) = \cot(\cot^{-1}(x))$$

$$\cot(y) = x$$



$$\cot(y) = \frac{x}{1}$$

$$\cot(y) = \frac{\text{adj}}{\text{opp}}$$

$$x^2 + 1^2 = \text{hyp}^2$$

$$\sqrt{x^2+1} = \text{hyp}$$

$$-\csc^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{-\csc^2 y}$$

$$\frac{d}{dx} (\cot^{-1}(x)) = \frac{-1}{x^2+1}$$

$$\frac{1}{-\csc^2 y} = -\frac{1}{\frac{1}{\sin^2 y}} = -\sin^2 y$$

$$-\left(\frac{\text{opp}}{\text{hyp}}\right)^2 = -\left(\frac{1}{\sqrt{x^2+1}}\right)^2 = \frac{-1}{x^2+1}$$

Check

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pg 374 Basic Differentiation Rules for Elementary Functions

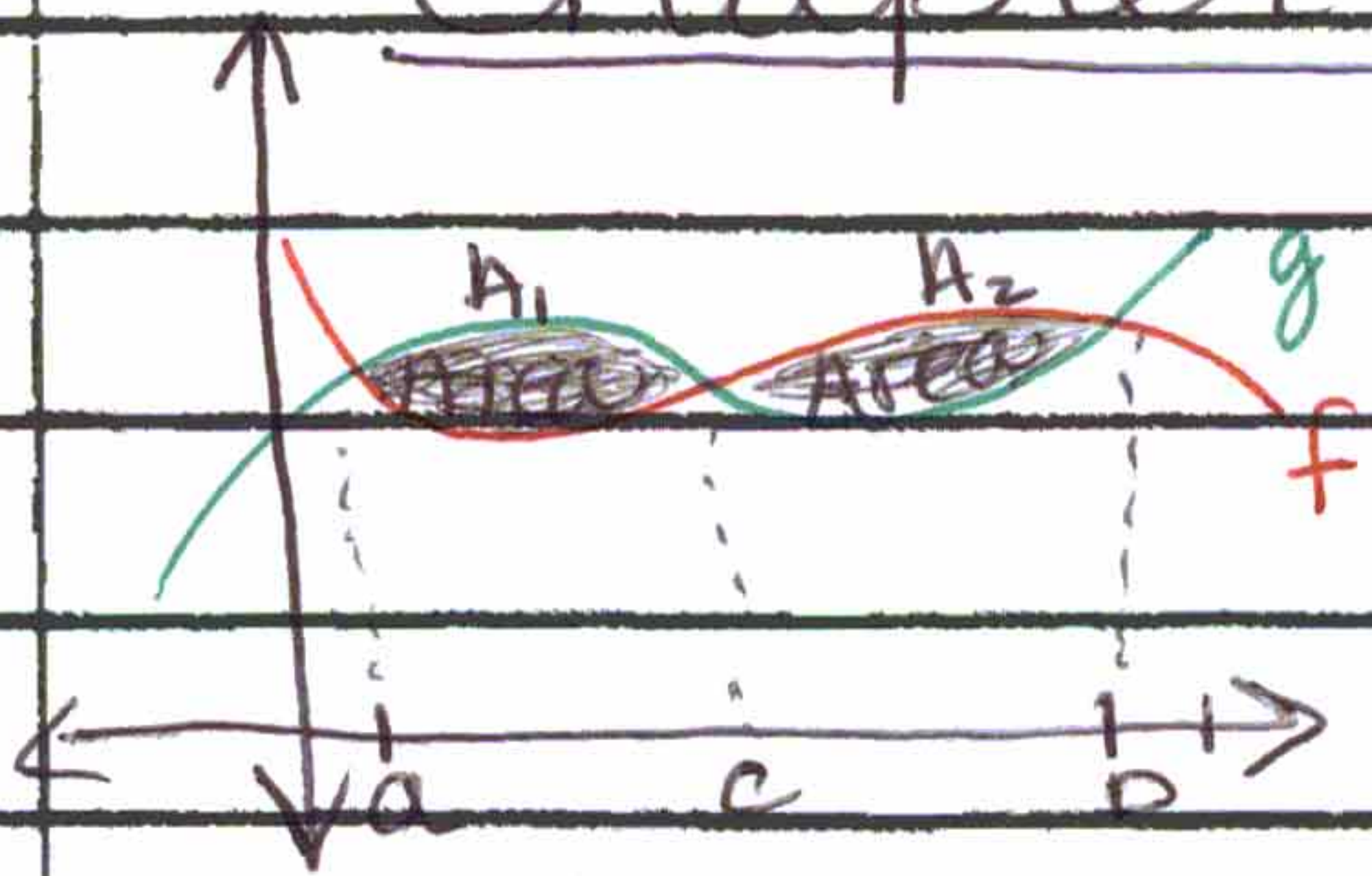
$$\int \frac{1}{x^2+1} dx = \cot^{-1} x$$

$$\cot^{-1}(x) = -\int \frac{1}{x^2+1}$$

esp. maintain chapter 5 for Calculus 2!

(not on final)

Chapter 7



$$\int_a^c [g(x) - f(x)] dx + \int_c^b [f(x) - g(x)] dx$$

$$A = A_1 + A_2$$

$$= \int_a^c [g(x) - f(x)] dx$$

5.2 #53

$$\int_1^2 \frac{1 - \cos \theta}{\theta - \sin \theta} d\theta = \int_{1-\sin 1}^{2-\sin 2} \left(\frac{1 - \cos \theta}{u} \right) \left(\frac{du}{1 - \cos \theta} \right)$$

let $u = \theta - \sin \theta$
 $\frac{du}{d\theta} = 1 - \cos \theta$

$$= \int_{1-\sin 1}^{2-\sin 2} \frac{1}{u} du = \left[\ln |u| \right]_{1-\sin 1}^{2-\sin 2}$$

$\frac{d\theta}{d\theta} = \frac{du}{1 - \cos \theta}$
 upper $\theta = 2$ lower $\theta = 1$

$u = 2 - \sin 2$ $u = 1 - \sin 1$

$$= \ln |2 - \sin 2| - \ln |1 - \sin 1|$$

acceptable for Boroids

$$\ln \frac{2 - \sin 2}{1 - \sin 1}$$

simplified

5.1 #85 $y = x \ln x$

$y' = 1 + \ln x$ ← find critical #s

$y' = (x) \frac{d}{dx}(\ln x) + (\ln x) \frac{d}{dx}(x)$

$y'' = \frac{d}{dx}(1 + \ln x)$

$y' = (x)(1/x) + (\ln x)(1)$

$y'' = 0 + 1/x$

$y' = 1 + \ln x$

$y'' = 1/x$ ← find P.P.I.

$\ln x = -1/x = -1$

$y''(1/2) = 0 \rightarrow$ concave up

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5.4 (#30) $y = e^{-x^2}$ so $y = e^u$

let $u = -x^2$ $\frac{d}{dx} y = \frac{d}{dx} (e^u)$

$\frac{du}{dx} = -2x$ $\frac{dy}{dx} = e^u \frac{du}{dx}$

$dx = \frac{du}{-2x}$ $\frac{dy}{dx} = e^{-x^2} (-2x)$

$\frac{d}{dx} y = \frac{d}{dx} (e^{-x^2})$

$= -2x \cdot e^{-x^2}$

performed method for now

or $\ln(y) = -x^2$

$\frac{d}{dx} [\ln(y)] = \frac{d}{dx} (-x^2)$

$y \cdot \frac{1}{y} \cdot \frac{dy}{dx} = -2x \cdot y$

$\frac{dy}{dx} = -2x \cdot e^{-x^2}$

logarithmic differentiation

If $y = \pi^x$ find $\frac{dy}{dx}$

$\ln(y) = \ln(\pi^x)$

$\ln(y) = x \ln(\pi)$

$\frac{d}{dx} [\ln(y)] = \frac{d}{dx} [x \cdot \ln(\pi)]$

$y \cdot \frac{1}{y} \cdot \frac{dy}{dx} = \ln(\pi) \cdot y$

$\frac{dy}{dx} = \ln(\pi) \cdot \pi^x$

Variable exponent use natural log

let the function tell you which approach

5.4 #90

$\int \frac{e^{2x}}{1+e^{2x}} dx$ let $u = 1+e^{2x}$

$= \int \frac{e^{2x}}{u} \frac{du}{2e^{2x}}$ $\frac{du}{dx} = (e^{2x}) \frac{d}{dx} (2x)$

$= \int \frac{1}{u} \frac{du}{2}$ $\frac{du}{dx} = 2e^{2x}$

$= \frac{1}{2} \ln|u| + C$ $dx = \frac{du}{2e^{2x}}$

$= \frac{1}{2} \ln|1+e^{2x}| + C$