Study Guide for Final Exam

Solve and state your result on a solution set. Round answers to nearest tenth, if necessary.

1. \[-[x - (4x - 7)] = 3 - (x - 6)\]
2. \[0.6(14x - 8000) = -0.4(20x + 12,000) + 20.6x\]
3. \[\frac{1}{3}x = 10 - \frac{1}{2}x\]
4. \[3(6 - 4x) = 2(-6x + 9)\]
5. \[|2x - 9| = 18\]
6. \[14 = \left| 2 - \frac{5}{2}m \right|\]

Solve each inequality; graph the solution set on a number line, and state this set in interval notation.

7. \[-3(-2x + 12) < -4(x + 2) - 6\]
8. \[\frac{3y - 6}{2} > \frac{2y + 5}{6}\]
9. \[|2x - 6| + 5 \leq 2\]
10. \[\left| \frac{x}{2} + 4 \right| - 3 \geq 2\]

Evaluate \(f(x) = -7x + 3\) at the indicated values:

11. \[f(-3)\]
12. \[f\left(\frac{-2}{3}\right)\]

Evaluate \(g(x) = 3x^2 - 2x + 5\) at the indicated values:

13. \[g(0)\]
14. \[g(-2)\]

Determine which of the relations are also functions. Give the domain and range of each relation.

15. \{(2,5), (3,7), (4,9), (5,11)\}
16. \{(-1,1), (1,-1), (2,4), (2,-4), (3,9), (3,-9)\}
19. Consider the graph below:

20. Simplify.
\[
(16t^3s^2 + 8t^2s^3 + 9ts^4) - (-18ts^4 + 3t^2s^3 - 24t^3s^2)
\]

\[
\left[ (9b^3 - 4b^2 + 3b + 2) - (-2b^3 - 3b^2 + b) \right] - (8b^3 + 6b + 4)
\]

22. Divide.
\[
\frac{-27x^4y^4 + 36x^3y^2 - 6x^3 - 26x^2y^2 + 2y^3}{-3x^2y}
\]

23. Factor completely.
\[
-17x^2y - 34x^2y^3
\]

24. \[
(3x - 2)(4x^6 - 5x^4 + 6x^3 - 7x + 8)
\]

25. \[
(w - 3)^3
\]

26. Solve each equation. State your solution as a solution set.
\[
4a(2a + 3) = 36
\]

\[
2k(k + 3) = (3k + 1)(k + 3)
\]

\[
x^3 + 12x = 7x^2
\]

40. The floor of a shed has an area of 54 square feet. Find the length and width if the length is 3 feet less than twice its width. (Draw a picture, define a variable, create an
41. Two cars leave an intersection. One travels north; the other travels east. When the car traveling north had gone 24 miles, the distance between the cars was four more than three times the distance traveled by the car heading east. Find the distance between the cars at that time. (Draw a picture, define a variable, create an equation, solve using algebra, and answer in a sentence.)

42. An object is propelled from a height of 48 feet with an initial velocity of 32 feet per second, its height is given by the equation \( h = -16t^2 + 32t + 48 \). After how many seconds is the height 60 feet? (Draw a picture, define a variable, create an equation, solve using algebra, and answer in a sentence.)

43. Determine the value or values of the variable where \( \frac{x+4}{2x-6} \) is defined.

44. Determine the value or values of the variable where \( \frac{x+5}{2x^2-x-3} \) is defined.

45. What is the domain of a rational function? Give two examples of rational functions and state their domains.

Simplify:

46. \( \frac{2z-4}{8-4z} \)

47. \( \frac{x^2-2x-24}{6-x} \)

Multiply:

48. \( \frac{a^2 + 3a - 10}{2a} \cdot \frac{a^2 - 3a}{a^2 - 5a + 6} \)

49. \( \frac{2x^2 - 3xy - 2y^2}{3x^2 - 4xy + y^2} \cdot \frac{3x^2 - 2xy - y^2}{x^2 + xy - 6y^2} \)

Divide:

50. \( \frac{2b^2 + 9b + 4}{b^2 + 7b + 12} + \frac{2b^2 - b - 1}{(b + 3)^2} \)

51. \( \frac{20w^2 - 17w + 3}{25w^2 - 9} + \frac{4w^2 - 9w + 2}{5w^2 - 7w - 6} \)

52. Explain how the LCD is used in a different way when adding and subtracting rational expressions compared to solving equations with rational expressions. Show one example of each technique.

Add or subtract:
53. \[ \frac{2}{x+1} - \frac{3x}{3x+3} + \frac{1}{2x+2} \]

54. \[ \frac{4x}{x^2 - 9} - \frac{2}{x + 3} \]

Simplify:
\[ \frac{7}{x} + 3 \]
\[ \frac{9}{x} - \frac{1}{2x^2} - \frac{1}{3} \]

55. \[ \frac{3x + 6}{2x^2 + x - 6} + \frac{3x}{2x - 3} \]

56. \[ \frac{9m + 2}{3m^2 - 2m - 8} + \frac{7}{3m^2 + m - 4} \]

57. \[ \frac{1}{y - 1} + 1 \]
\[ \frac{1}{y + 1} - 1 \]

58. Solve each equation. State your result in a solution set.

59. \[ \frac{x}{3} - \frac{3x}{4} = \frac{1}{12} \]

60. \[ \frac{2}{x - 3} - \frac{4}{x + 3} = \frac{8}{x^2 - 9} \]

61. \[ \frac{b}{2} - \frac{4}{b} = -\frac{7}{2} \]

62. \[ \frac{5y - 3}{2y} = \frac{10y + 3}{4y + 1} \]

63. Solve \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \) for \( q \).

64. Two trains leave at the same time going in opposite directions. One train travels 15 mph faster than the other. In 6 hours the trains are 630 miles apart. Find the speed of each. (Draw a picture, set-up a chart, define a variable, create an equation, solve using algebra, and answer in sentence.)

65. An experienced roofer can roof a house in 26 hours. A beginning roofer needs 39 hours for the same job. Find how long it takes for the two to do the job working together? (Set-up an organizational chart structure, define a variable, create an equation, solve using algebra, and answer in a sentence.)

66. Find the length of the side indicated by \( x \). (Answer in a sentence.)

67. Graph each function and give its domain and range
\[ f(x) = \sqrt{x + 3} \]

68. \[ g(x) = \sqrt[3]{x - 1} \]
Simplify.

69. \( \sqrt[3]{\frac{1000}{27}} \)

70. \(-16w^7f^2 \cdot 4w^{-8}f^3\)

71. \( \left( \frac{250g^{-3}h^5}{2g^{-2}h^2} \right)^{2/3} \)

72. \( \left( \frac{b^{-3/2}}{c^{-5/3}} \right)^2 \left( b^{-1/4} \cdot c^{-1/3} \right)^{-1} \)

73. \( \left( \frac{2^{-2}w^{-3}x^{-5}y^8}{w^{3/4}x^{-1/4}} \right)^{-3} \)

74. \( 3\sqrt{25ab^2} \cdot 3\sqrt{5a^7b^{14}} \)

75. \( 2\sqrt[3]{x^4y^5} \left( \frac{3}{\sqrt[3]{27x^{12}y^4}} - 3\sqrt[3]{64xy^9} \right) \)

76. \( \frac{3\sqrt{5}}{\sqrt{x^3}} \)

77. \( \frac{3\sqrt{5t^2}}{\sqrt[3]{3w^2}} \)

78. \(-\frac{5}{\sqrt{24}} \)

79. \( (\sqrt{7} + \sqrt{8})(\sqrt{7} - \sqrt{8}) \)

80. \( \frac{4}{\sqrt{5} - 7} \)

81. \( \frac{-1}{3\sqrt{2} - 2\sqrt{7}} \)

82. \( \sqrt{24} - 6\sqrt{54} \)

83. \( 2\sqrt{50} - 3\sqrt{125} + \sqrt{98} \)

84. \( x^3\sqrt{64x^5y^2} - x^2\sqrt[3]{27x^2y^2} + 2\sqrt[3]{8x^8y^2} \)

85. \( (8\sqrt{6} - 7)(5\sqrt{6} - 3) \)

86. \( (4\sqrt{5} - 6)^2 \)

Solve each equation. Write the answer in a solution set.

87. \( \sqrt[3]{x - 2} - 3 = 0 \)

88. \( 2x + \sqrt{x} + 1 = 8 \)

89. \( \sqrt{y + 5} = 2 - \sqrt{y - 4} \)

90. Ms. Song Tran places an extension ladder against her house. The base of the ladder is 2 meters from the house and the ladder rests against the house 6 meters above the ground. How far is her ladder extended? (Draw a picture, define a variable, create an equation, solve using algebra, and answer in a sentence. Round to the nearest hundredths.)

Find the length of the unknown side:

91. 

\[
\begin{align*}
9m & \\
11m & \\
\end{align*}
\]

\[ b \]
Simplify, and write in the form $a + bi$ if possible.

92. $\sqrt{-72}$

93. $[7 + 2i] + (-4 - i) - (2 - 5i)$

94. $\sqrt{-8(\sqrt{2} - \sqrt{-2})}$

95. $(-7 + 3i)(9 - 7i)$

Solve the following equations, and write your solution as a set.

99. $(y - 3)^2 = 45$

100. $(2x - 3)^2 = 8$

Solve by **COMPLETING THE SQUARE**, and write your solution as a set.

101. $b^2 = 4b + 32$

102. $x^2 + 6x + 2 = 0$

Solve using the **QUADRATIC FORMULA**, and write your solution as a set.

103. $3w^2 - 4w = -5$

104. $b^2 = 4b + 32$

105. $\frac{1}{2}y^2 = y - \frac{1}{2}$

106. Julie Hildebrand, a long distance runner, starts jogging at her house. She jogs 6 miles and then turns around and jogs back to her house. The first part of her jog is mostly uphill, so her speed averages 2 miles per hour less than her returning speed. If the total time she spent jogging was $1 \frac{3}{4}$ hours, find her going and her speed returning. (Set-up an organizational chart structure, define a variable, create an equation, solve using algebra, and answer in a sentence.)

107. Two ships leave port at the same time, one heading due south and the other due east. Several hours later, they are 71 miles apart. If the ship traveling south traveled 21 miles farther than the other ship, how many miles did they each travel? (Draw a diagram of the situation, define a variable, create an equation, solve using algebra, and answer in a sentence.)

108. A rocket is launched from the top of an 80-foot cliff with an initial velocity of 120 feet per second. The height of the rocket $h(t)$ after $t$ seconds is given by the function $h(t) = -16t^2 + 120t + 80$. How long after the rocket is launched will it **strike the ground**? (Answer in sentence.)

109. $4x^2 = -12x - 9$

Use the discriminant to determine the number and types of solution for the following equation.

109. $4x^2 = -12x - 9$
Use “u-substitution” to solve the following equation, and write the solution as a set.

110. \( w^4 + 2w^2 = 8 \)

111. \( 2x^2 + 3x + 2 = 0 \)

Sketch each graph.

112. \( f(x) = -(x - 2)^2 - 6 \)
   (a) vertex = (__, __)
   (b) axis of symmetry = ______
   (c) opens wider/narrower
   (d) opens up/down
   (e) \( x \)-intercept = (__, __)
   (f) \( y \)-intercept = (__, __)
   (g) Domain = ______
   (h) Range = ______

113. \( h(x) = 2x^2 + 4x + 5 \)
   (a) vertex = (__, __)
   (b) axis of symmetry = ______
   (c) opens wider/narrower
   (d) opens up/down
   (e) \( x \)-intercept = (__, __)
   (f) \( y \)-intercept = (__, __)
   (g) Domain = ______
   (h) Range = ______

114. If Rheam Gaspar throws a ball upward with an initial speed of 32 feet per second then its height \( h(t) \) in feet after \( t \) seconds is given by the function \( h(t) = -16t^2 + 32t \). Find the maximum height of the ball. (Solve using algebra, and answer in a sentence.)

Determine whether each function is a one-to-one function.

115. \( \{(1, 2), (4, 3), (3, 4), (6, 6)\} \)

116.
Each function is one-to-one. Find the inverse of each function and graph the function and its inverse on the same set of axes.

117. \( f(x) = \frac{1}{2}x - 4 \) 

118. \( f(x) = -4x + 1 \)

119. Discuss the purpose of the vertical line test and the horizontal line test. Give an example of each type test.

Find the value of each expression:

120. \( \log_2 16 \)

121. \( \log_8 \left( \frac{1}{2} \right) \)

122. An accidental spill of 75 grams of radioactive material in a local stream has led to the presence of radioactive debris decaying at a rate of 4% each day. Find the percent of this debris still remaining after 14 days. Use \( y = 75(2.7)^{-0.04t} \). (Answer in a sentence.)

123. Find the amount owed at the end of 5 years if $3,000 is loaned at a rate of 10% compounded quarterly. Use \( A = P \left( 1 + \frac{r}{n} \right)^{nt} \). (Answer in a sentence.)

Write each expression as a sum or difference of multiples of logarithms. Assume that variables represent positive numbers.

124. \( \log_2 \left( \frac{x^3}{y} \right) \)

125. \( \log_5 x^3 (x + 1) \)

Write each as the logarithm of a single expression. Assume that variables represent positive numbers.

126. \( \log_6 18 + \log_6 2 - \log_6 9 \)

127. \( \log_9(4x) - \log_9(x - 3) + \log_9(x^3 + 3) \)

128. \( 2 \log_5 x + \frac{1}{3} \log_5 x - 3 \log_5(x + 5) \)

Solve each equation. Write the answer in a solution set.
129. \( \log_8 x = \frac{1}{3} \)

130. \( \log_x 27 = 3 \)

131. \( \log_5 \frac{1}{125} = x \)

132. \( 5^x = 625 \)

133. \( 32^x = 4 \)

134. \( 4^{3x-7} = 32^{2x} \)

135. \( \frac{1}{27} = 3^{2x} \)

136. \( \log_4 (2x + 2) = 3 \)

137. \( \log_2 (a + 4)^2 = 4 \)

138. \( \log_6 (x + 2) - \log_6 x = 2 \)

139. \( \log_3 (x - 8) + \log_3 x = 2 \)