1. **Definition of Improper Integrals with Infinite Integration Limits:**

1. If \( f \) is continuous on \([a, \infty)\), then

\[
\int_a^\infty f(x) \, dx = \lim_{k \to \infty} \int_a^k f(x) \, dx
\]

2. If \( f \) is continuous on \((-\infty, b]\), then

\[
\int_{-\infty}^b f(x) \, dx = \lim_{k \to -\infty} \int_k^b f(x) \, dx
\]

3. If \( f \) is continuous on \((-\infty, \infty)\) and \( c \) is any real number, then

\[
\int_{-\infty}^\infty f(x) \, dx = \lim_{k \to -\infty} \int_k^c f(x) \, dx + \lim_{j \to \infty} \int_c^j f(x) \, dx
\]

We say that the improper integral **converges** if the limit exists (the limit is a finite number). We say that the improper integral **diverges** if the limit does not exist (the limit goes to \( \pm \infty \)).

2. **Definition of Improper Integrals with Infinite Discontinuities:**

1. If \( f \) is continuous on \([a, b]\) and has an infinite discontinuity at \( b \), then

\[
\int_a^b f(x) \, dx = \lim_{k \to b^-} \int_a^k f(x) \, dx
\]

2. If \( f \) is continuous on \((a, b]\) and has an infinite discontinuity at \( a \), then

\[
\int_a^b f(x) \, dx = \lim_{k \to a^+} \int_k^b f(x) \, dx
\]

3. If \( f \) is continuous on \([a, b]\), except for some \( c \) in \((a, b)\) at which \( f \) has an infinite discontinuity, then

\[
\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx = \lim_{k \to c^-} \int_a^k f(x) \, dx + \lim_{j \to c^+} \int_j^b f(x) \, dx
\]

3. **Theorem: A Special Type of Improper Integral**

\[
\int_1^\infty \frac{1}{x^p} \, dx = \begin{cases} 
\frac{1}{p-1} & \text{if } p > 1 \\
\text{diverges} & \text{if } p \leq 1
\end{cases}
\]