1. **Definition:** (Taylor Polynomial) The $n$-th degree Taylor Polynomial of the function $f$ at the point $x = a$ is given by

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} (x-a)^k$$

$$= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} (x-a)^2 + \frac{f'''(a)}{3!} (x-a)^3 + \frac{f^{(4)}(a)}{4!} (x-a)^4 + \cdots + \frac{f^{(n)}(a)}{n!} (x-a)^n$$

2. **Definition:** (Mclaurin Polynomial) The $n$-th degree Mclaurin Polynomial of the function $f$ is a Taylor Polynomial when $a = 0$, which is given by

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k$$

$$= f(0) + f'(0)x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \frac{f^{(4)}(0)}{4!} x^4 + \cdots + \frac{f^{(n)}(0)}{n!} x^n$$

3. **Theorem:** Taylor’s Theorem (Taylor’s Formula with Remainder):

Let $f$ be a function whose $(n+1)$-st derivative $f^{(n+1)}(x)$ exists for each $x$ in an open interval $I$ containing $a$. Then, for each $x$ in $I$,

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} (x-a)^k + R_n(x)$$

ie.

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!} (x-a)^2 + \frac{f'''(a)}{3!} (x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!} (x-a)^n + R_n(x)$$

where the remainder (or error) $R_n(x)$ is given by the formula

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1}, \text{ and } c \text{ is some point between } x \text{ and } a.$$  

Furthermore,

$$|R_n(x)| = \left| \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1} \right| \leq \frac{|x-a|^{n+1}}{(n+1)!} \max |f^{(n+1)}(c)|$$

where $c$ is some point between $a$ and $x$. 
