1. **Definition: (Power Series)** If $x$ is a variable, then an infinite series of the form

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n + \cdots$$

is called a power series. More generally, an infinite series of the form

$$f(x) = \sum_{n=0}^{\infty} a_n (x - c)^n = a_0 + a_1 (x - c) + a_2 (x - c)^2 + a_3 (x - c)^3 + \cdots + a_n (x - c)^n + \cdots$$

is called a power series centered at $c$, where $c=$constant.

**NOTE:** To simplify the notation for power series, we need to agree that $(x - c)^{0} = 1$, even if $x = c$. (ie. $0^0 = 1$)

2. **Definition: (Domain of a Series)** Let $f(x)$ be a power series centered at $c$. Then we way that the domain of $f(x)$,

$$f(x) = \sum_{n=0}^{\infty} a_n (x - c)^n,$$

is set of all $x$ for which the power series converges. The set of all $x$ values for which the power series converge is called the interval of convergence.

3. **Theorem (Convergence of a Power Series):**

Let $R > 0$ be a real number. The convergence set for a power series $\sum_{n=0}^{\infty} a_n (x - c)^n$ is always an interval of one of the following three types:

1. A single point $x = c$. In this case, the Radius of Convergence is $R=0$.

2. An interval $(c - R, c + R)$ (ie. $|x - c| < R$), plus possible one or both endpoints. In this case, the Radius of Convergence is $R$ itself.

3. The whole real line. In this case, the Radius of Convergence is $R = \infty$. 
Furthermore, a power series $\sum_{n=0}^{\infty} a_n(x - c)^n$ converges absolutely on the interior of its interval of convergence, and outside of the given interval, the power series diverges.

4. **Theorem (Properties of Functions Defined by Power Series):**

Suppose that $f(x)$ is the sum of a power series on an interval $(c - R, c + R)$. That is

$$f(x) = \sum_{n=0}^{\infty} a_n(x - c)^n = a_0 + a_1(x - c) + a_2(x - c)^2 + a_3(x - c)^3 + \cdots + a_n(x - c)^n + \cdots$$

Then $f(x)$ is differentiable (hence continuous) on the interval $(c - R, c + R)$ and

1. $$f'(x) = \sum_{n=0}^{\infty} \frac{d}{dx} [a_n(x - c)^n]$$
   $$= \sum_{n=1}^{\infty} n a_n (x - c)^{n-1}$$
   $$= a_1 + 2a_2(x - c) + 3a_3(x - c)^2 + \cdots$$

2. $$\int f(x) \, dx = \int \sum_{n=0}^{\infty} a_n(x - c)^n \, dx$$
   $$= \sum_{n=0}^{\infty} a_n \int (x - c)^n \, dx$$
   $$= \sum_{n=0}^{\infty} a_n \frac{(x - c)^{n+1}}{n + 1} + C$$
   $$= C + \sum_{n=0}^{\infty} a_n \frac{(x - c)^{n+1}}{n + 1}$$
   $$= C + a_0(x - c) + a_1 \frac{(x - c)^2}{2} + a_2 \frac{(x - c)^3}{3} + \cdots$$

The radius of convergence of the $f'(x)$ and $\int f(x) \, dx$ is the same as that of the original power series. However, the interval of convergence may be different at the end points.