$\boxed{P3}$ Functions & their Graphs

#4 $f(x) = \sqrt{x+3}$

(a) $f(-2) = \sqrt{(-2)+3}$
$f(-2) = \sqrt{1}$
$f(-2) = 1$

(b) $f(6) = \sqrt{(6)+3}$
$f(6) = \sqrt{9}$
$f(6) = 3$

(c) $f(-5) = \sqrt{(-5)+3}$
$f(-5) = \sqrt{-2}$
$f(-5) =$ not a real number

★ $\sqrt{-1} = i$ ← imaginary unit

(d) $f(x+\Delta x) = \sqrt{(x+\Delta x)+3}$
$f(x+\Delta x) = \sqrt{x+\Delta x+3}$

If $g(x) = x^2$, find $\dfrac{g(x+\Delta x) - g(x)}{\Delta x}$ & simplify

$$\frac{g(x+\Delta x)-g(x)}{\Delta x} = \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

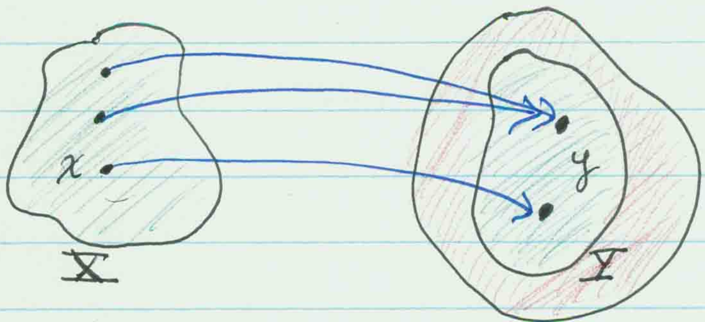
$$= \frac{\Delta x\,(2x + \Delta x)}{\Delta x}$$

$$= 2x + \Delta x$$

Definition of Function:

Let X & Y be two nonempty sets. A function from X into Y is a relation that associates with each element of X exactly one element of Y.

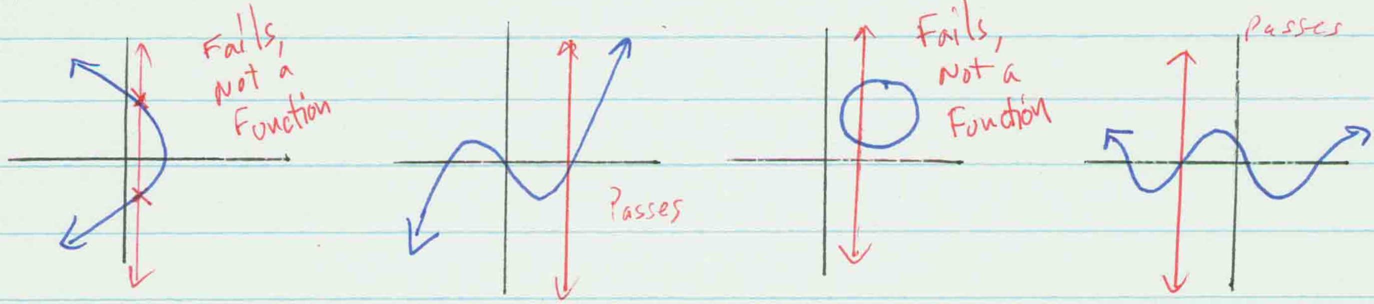
The set X is called the domain of the function. For each element in X, the corresponding element y in Y is called the value of the function at x, or the image of x. The set of all images of the elements in the domain is called the range of the function.



Since there may be some elements in Y that are not the image of some x in X, it follows the the range of a function may be a subset of Y.
Not all relations between two sets are functions.

Vertical Line Test:

A vertical line can intersect the graph of a function at most most once.



Find the domain of $f(x) = \sqrt{x^2 - 3x + 2}$

$x^2 - 3x + 2 \geq 0$ for $f(x)$ to real number values

solve:

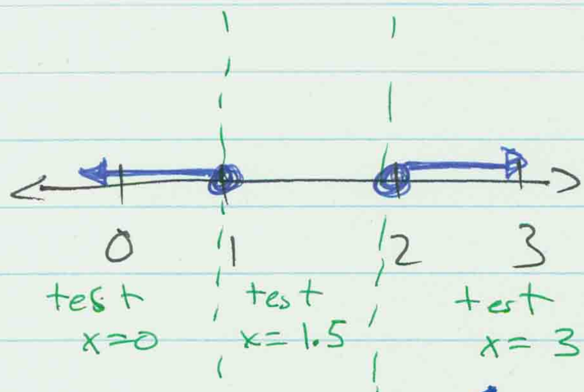
$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

either

$x - 1 = 0$, or $x - 2 = 0$

$x = 1$ $\qquad$ $x = 2$



Test

$x = 0$: $(0)^2 - 3(0) + 2 \geq 0$

$2 \geq 0$, True

$x = 3$: $(3)^2 - 3(3) + 2 \geq 0$

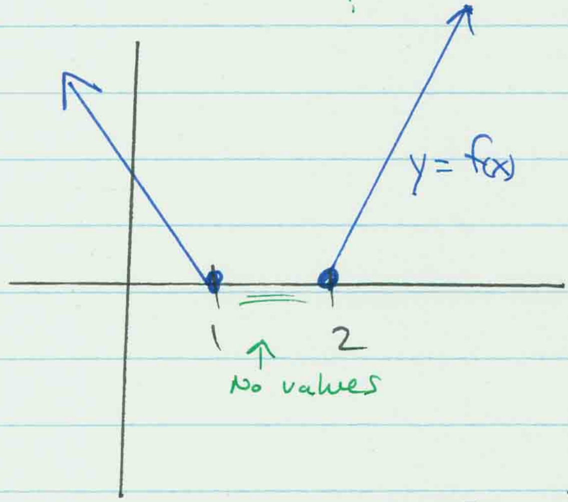
$9 - 9 + 2 \geq 0$

$2 \geq 0$, True

$x = 1.5$: $(1.5)^2 - 3(1.5) + 2 \geq 0$

$2.25 - 4.5 + 2 \geq 0$

$-0.25 \geq 0$, False



The domain is $\{x \mid x \leq 1 \text{ or } x \geq 2\} = (-\infty, 1] \cup [2, \infty)$.

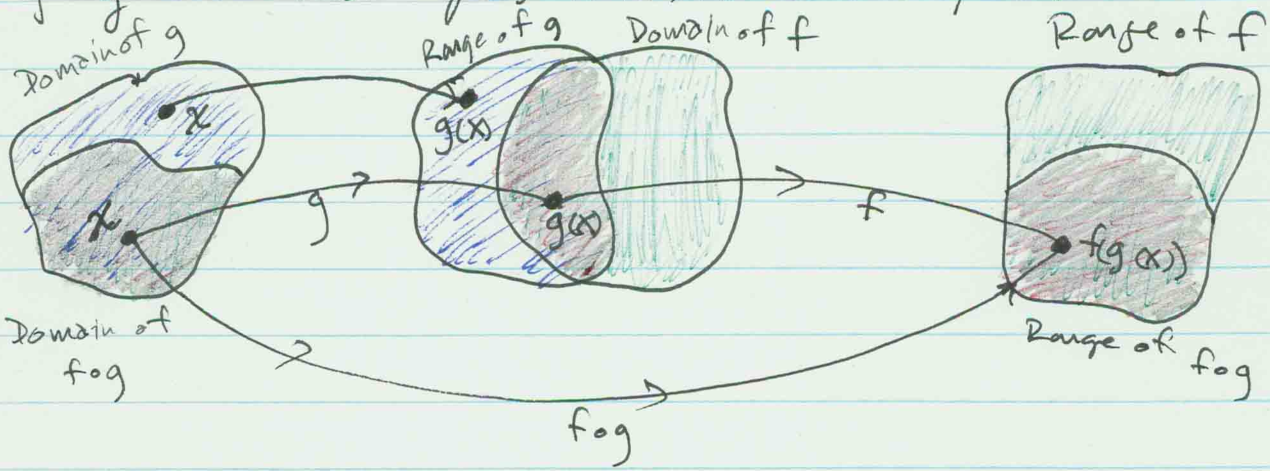
P3

Composite function

Given two functions f and g, the composite function, denoted by $f \circ g$, is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.



Given that $f(x) = x^2 + 3x - 1$ & $g(x) = 2x + 3$

(a) $(f \circ g)(x) = f(g(x)) = (g(x))^2 + 3(g(x)) - 1$,

$= (2x+3)^2 + 3(2x+3) - 1$

$= 4x^2 + 12x + 9 \quad + 6x + 9 - 1$

$(f \circ g)(x) = 4x^2 + 18x + 17$

(b) $(g \circ f)(x) = g(f(x)) = 2(f(x)) + 3$,

$= 2(x^2 + 3x - 1) + 3$

$= 2x^2 + 6x - 2 + 3$

$(g \circ f)(x) = 2x^2 + 6x + 3$

$\boxed{P.3}$

Even & ODD Functions:

Even functions are symmetric about the y-axis.
ODD functions are symmetric about the origin.

- The function $y=f(x)$ is even if $f(-x)=f(x)$
- The function $y=f(x)$ is odd if $f(-x)=-f(x)$

Test for Even & ODD Functions

(a) $f(x)=x^2-5$

$$f(-x)=(-x)^2-5$$
$$f(-x)=x^2-5$$
$$f(-x)=f(x)$$

, So, $f(x)=x^2-5$ is even.

(b) $h(x)=5x^3-x$

$$h(-x)=5(-x)^3-(-x)$$
$$h(-x)=-5x^3+x$$
$$h(-x)=-(5x^3-x)$$
$$h(-x)=-h(x)$$

. So, $h(x)=5x^3x$ is odd.