Section 3.1 Meaning and Properties of Fractions

1. Definition of a Fraction: A fraction is any number that can be put in the form $\dfrac{a}{b}$ where a and b are integers and b is not 0. The numerator of the fraction is "a" and the denominator is "b". A proper fraction is a fraction in which the numerator is less than the denominator. An improper fraction is a fraction in which the numerator is greater than or equal to the denominator.
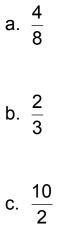
Example: Answer each of the following.
a. Give an example of a proper fraction.
b. Give an example of an improper fraction.
c. What are the integers?
d. Is $\dfrac{7}{7}$ a proper fraction or an improper fraction?

2. Equivalent Fractions: Fractions that represent the same number are said to be equivalent.

Example: For each fraction below, name an equivalent fraction.

a. $\dfrac{4}{8}$

b. $\dfrac{2}{3}$

c. $\dfrac{10}{2}$

3. Property One for Fractions: If a, b and c are integers and b and c are not 0, then it is true that

$$\frac{a}{b} = \frac{a \bullet c}{b \bullet c}$$

This property is often used to rewrite a given fraction as an equivalent fraction with a specific denominator. This is particularly useful when rewriting a several fractions with a common denominator.

Note: Portions of this document are excerpted from the textbook *Prealgebra,* 7th ed. by Charles McKeague

Example: Rewrite each fraction as an equivalent fraction with the given denominator.

a. $\dfrac{3}{4} = \dfrac{}{20}$

b. $\dfrac{5}{7}$, denominator of 35

c. $\dfrac{9}{11} = \dfrac{}{33}$

d. $\dfrac{8}{10}$, denominator of 25

4. Property Two for Fractions: If a, b and c are integers and b and c are not 0, then it is true that

$$\dfrac{a}{b} = \dfrac{a \div c}{b \div c}$$

This property is often used to reduce a given fraction to lowest terms. To reduce to lowest terms, the number "c" is the greatest common factor for the numerator and denominator.

Example: Reduce each fraction to lowest terms.

a. $\dfrac{30}{45}$

b. $\dfrac{10}{14}$

c. $\dfrac{22}{33}$

d. $\dfrac{8}{10}$

5. The Number 1 and Fractions: If a is any number, then it is true that

$$\frac{a}{1} = a \quad \text{and} \quad \frac{a}{a} = 1.$$

Example: Simplify each expression.

a. $\dfrac{7}{1}$

b. $\dfrac{16}{16}$

c. $\dfrac{-3}{-3}$