Section 4.4 Linear Equations in One Variable

1. Definition of a Linear Equation in One Variable: A linear equation in one variable contains only one variable and that variable is always raised to the first power and never appears in a denominator.

   Example 1: Identify which of the following are linear equations in one variable.
   a. $2x - 7 = 4x - 15$
   b. $x^2 - 2x + 3 = 0$
   c. $2x - 5 = x^3$
   d. $\frac{2}{x} + 1 = 15$  
   e. $2x + 4 = \sqrt{x}$  (Note: $\sqrt{x} = x^{1/2}$)

2. Steps to Solve a Linear Equation:
   - Simplify each side of the equation as much as possible. Combine similar terms and use the distributive property.
   - Use the addition property of equality to get all variable terms on one side of the equation and all constant terms on the other, and then combine similar terms. A variable term is any term that contains the variable. A constant term is any term that contains only a number.
   - Use the multiplication property of equality to get the coefficient of the variable term to be 1.
   - Check your solution in the original equation.
Example 2: Solve the given equations.

a. \(3y + 5 = 9y + 8\)

b. \(10a + 3 = 4(a - 1) + 1\)

3. Solving Equations That Contain Fractions: If the equation contains only one fraction, you may be able to solve it easily by applying the Addition and/or Multiplication Properties. If the equation has several fractions, it is easier to use the method of clearing fractions to solve the equation. In this method, you find the LCD for all of the fractions in the equation, but instead of adjusting all of the fractions so that they have the LCD, you multiply both sides of the equation by the LCD, using the distributive property if needed. The resulting equations will no longer contain any fractions.
Example 3: Solve the given equations by clearing fractions.

a. \( \frac{x}{2} - \frac{x}{4} = 3 \)

b. \( \frac{3}{x} - \frac{4}{5} = -\frac{1}{5} \)

Practice Problems. Solve each equation. Show all steps.

a. \( 6y + 9 = 4y - 3 \)
b. \[5(2x - 4) + 8 = 38\]

c. \[3x + \frac{1}{2} = \frac{1}{4}\]

d. \[\frac{1}{2}x + \frac{4}{3} = -\frac{2}{3}\]

Answers to Practice Problems:

a. \(-6\); b. \(5\); c. \(-\frac{1}{12}\); d. \(-4\)

Note: Portions of this document are excerpted from the textbook *Prealgebra, 7th ed.* by Charles McKeague