Section 5.7 Square Roots and the Pythagorean Theorem

1. Definition of Square Root: The square root of a positive number a, written $\sqrt{a}$, is the number we square to get a. In symbols:

$$\text{If } \sqrt{a} = b \text{ then } b^2 = a.$$

A table of common square roots is given below.

Table of Common Square Roots

Statement	In Words	Reason
$\sqrt{0} = 0$	The square root of 0 is 0	Because $0^2 = 0$
$\sqrt{1} = 1$	The square root of 1 is 1	Because $1^2 = 1$
$\sqrt{4} = 2$	The square root of 4 is 2	Because $2^2 = 4$
$\sqrt{9} = 3$	The square root of 9 is 3	Because $3^2 = 9$
$\sqrt{16} = 4$	The square root of 16 is 4	Because $4^2 = 16$
$\sqrt{25} = ?$	The square root of 25 is ___	Because $?^2 = 25$
$\sqrt{36} = ?$	The square root of 36 is ___	Because $?^2 = 36$

Example 1: Simplify each of the following.

a. $\sqrt{49}$

b. $\sqrt{81} + \sqrt{16}$

c. $\sqrt{9+16}$

d. $\sqrt{\dfrac{16}{25}}$

2. Perfect Squares: A number whose square root is a whole number is called a perfect square. The perfect squares are:
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, …. Can you name the next two perfect squares?

3. Square Roots of Numbers That Are Not Perfect Squares:
If a number is not a perfect square, then its square root is a non-repeating, nonterminating decimal. Use your calculator to get a decimal approximation for the square root. Round your answer to the decimal place indicated in the directions.

Example 2: Use a calculator to approximate each of the following to three decimal places (nearest thousandth).

a. $\sqrt{37}$

b. $\sqrt{3} + \sqrt{26}$

c. $\sqrt{15 + 6}$

d. $\sqrt{\dfrac{5}{8}}$

4. The Pythagorean Theorem: A right triangle is a triangle that contains a 90° angle. The longest side, the one that is opposite the 90° angle, is the hypotenuse; and we use the letter c to denote it. The two shorter sides, the ones that form the 90° angle, are called the legs; and we use the letters a and b to denote them. The Pythagorean Theorem states that the hypotenuse is the square root of the sum of the squares of the legs. In symbols:

$$c = \sqrt{a^2 + b^2}$$

a

b
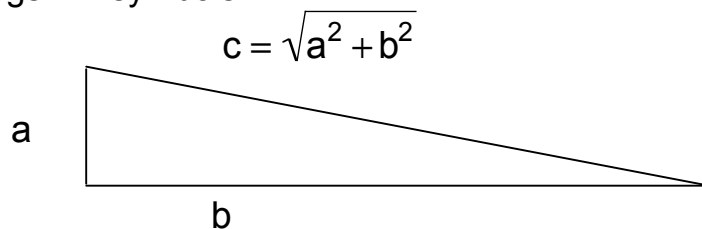
Example 3: Find the length of the hypotenuse in the given triangle. Use the proper format: write the formula, plug in the known values, find the hypotenuse. You may use your calculator to perform the calculations. Round your answer to the nearest hundredth. Be sure to use the correct units with your answer.

6 in.

9 in.
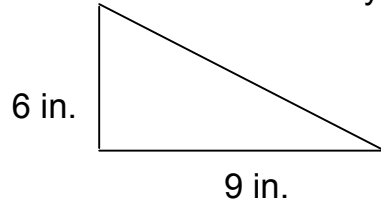
Practice Problems:

Simplify. Give exact answers.

a. $\sqrt{64}$

b. $\sqrt{25} + \sqrt{4}$

c. $\sqrt{64+36}$

d. $\sqrt{\dfrac{25}{49}}$

Simplfy using a calculator. Round your answer to three decimal places.

   e. $\sqrt{125}$

   f. $\sqrt{7} + \sqrt{52}$

   g. $\sqrt{23 + 11}$

   h. $\sqrt{\dfrac{15}{31}}$

Solve the following word problem, showing all steps. You must identify the quantity that the variable stands for, write an equation, solve the equation and then write out your answer in English words.

i. Find the hypotenuse of a right triangle that has legs of 34 inches and 57 inches. Give an exact answer, and then use a calculator to round your answer to three decimal places.

Answers to Practice Problems:
a. 8; b. 7; c. 10; d. 5/7; e. 11.180; f. 9.857; g. 5.831
h. 0.696; i. The hypotenuse is 66.370 inches long.