

9.2 Adding and Subtracting Polynomials

1. Vocabulary:

- A variable is a quantity represented by a letter.
- A polynomial is the sum of terms that contain variables raised to positive integer or zero powers and that have no variables in any denominator.
- A term is one of the addends in an addition expression. For example, in the expression $2x + 4$, the terms are $2x$ and 4 .
- The parts of each term that are multiplied are the factors of the term. For example, in the term $2x$ from the example above, the factors are 2 and x .
- Like terms have the same variable factors raised to the same powers. For example, in the expression $2x^2 + 3x + 7 + 3x^2 + 4x + 9$, the $2x^2$ and $3x^2$ are like terms, the $3x$ and $4x$ are like terms, and the 7 and 9 are like terms.

2. Adding Polynomials: To add two polynomials, use the commutative and associative properties of addition to rewrite the sum so that like terms are grouped, and then use the distributive property to combine like terms.

Example 1: Simplify.

$$\begin{aligned} \text{a. } (2x + 7) + (4x - 9) &= (2x + 7) + (4x + (-9)) \\ &= 2x + 7 + 4x + (-9) \\ &= 2x + 4x + 7 + (-9) \\ &= (2 + 4)x + (-2) \\ &= 6x - 2 \end{aligned}$$

$$\text{b. } (5x - 7) + (3x + 9)$$

$$c. (2x^2 + 7x + 4) + (4x^2 + 9x + 8)$$

3. Negating Polynomials: If there is a negative sign directly preceding the parenthesis surrounding a polynomial, the negative sign applies to each term inside the parenthesis. Use the distributive property to distribute the negation to each term inside the parenthesis. You may think of the negative preceding the parenthesis as a -1 , and use the rules for multiplying signed numbers.

Example 2: Simplify.

$$\begin{aligned} a. -(5x + 7) &= -1(5x + 7) \\ &= (-1)(5x) + (-1)(7) \\ &= -5x + (-7) \\ &= -5x - 7 \end{aligned}$$

$$b. -(3x + 6)$$

$$c. -(3x - 7)$$

$$d. -(5x^2 + 7x - 6)$$

4. Subtracting Polynomials: To subtract two polynomials, change the subtraction to addition of the opposite and then add.

Example: Simplify.

$$\begin{aligned} \text{a. } (2x + 3) - (5x + 7) &= (2x + 3) + (-1)(5x + 7) \\ &= 2x + 3 + (-1)(5x) + (-1)(7) \\ &= 2x + 3 + (-5x) + (-7) \\ &= 2x + (-5x) + 3 + (-7) \\ &= (-3x) + (-4) \\ &= -3x - 4 \end{aligned}$$

$$\text{b. } (3x + 5) - (7x + 2)$$

$$\begin{aligned} \text{c. } (7x + 8) - (6x - 9) &= (7x + 8) + (-1)(6x - 9) \\ &= (7x + 8) + (-1)(6x + (-9)) \\ &= 7x + 8 + (-1)(6x) + (-1)(-9) \\ &= 7x + 8 + (-6x) + (9) \\ &= x + 17 \end{aligned}$$

$$\text{e. } (3x^2 + 5x - 5) - (7x^2 - 2x - 4)$$

4. Evaluating Polynomials: To find the value of a polynomial at a given value of the variable, substitute the value of the variable into the polynomial everywhere the variable appears.

Example: Evaluate the given polynomial at the given value of the variable.

a. $2x - 7$ at $x = -2$

b. $2x^2 + 7x - 5$ at $x = -3$

Practice Problems. Simplify each of the following:

a. $(2x - 7) + (4x - 9)$

b. $(2x^2 + 7x + 4) + (4x^2 + 9x + 8)$

c. $-(6x + 9)$

d. $-(5x^2 + 7x - 6)$

d. $(6x - 5) - (5x + 2)$

e. $(4x - 6) - (7x - 4)$

Evaluate the given polynomial at the given value of the variable.

f. $2x^2 - 5x - 8$ at $x = -2$

Answers to Practice Problems

a. $6x - 16$; b. $6x^2 + 16x + 12$; c. $-6x - 9$; d. $-5x^2 - 7x + 6$
e. $-3x - 2$; f. The value of the polynomial is 10.

Note: Portions of this document are excerpted from the textbook *Prealgebra*, 7th ed. by Charles McKeague