Math Fundamentals for Statistics II (Math 95)

Unit 3: Probability

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"The 'How' and 'Whys' Guys"

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3.1: Playing a New Game

Nothing better than getting your game-like skills put to the test early in a new unit. The game we've selected to start with is: (drum roll) ….

Roulette!

An American roulette wheel has 18 black numbers, 18 red numbers and 2 green numbers (0 and 00). The 36 numbers that are black or red are split evenly between even and odd (1 – 36). The picture above shows the way the wheel is set up; Roulette is relatively easy to play as a player puts down chips of different values as a bet. The house spins the wheel and drops a ball in.

NOTE: There are other roulette wheels as well, and one version has only one green number, 0. This type is sometimes called European, but the 2-green number wheels are also used in Europe. There is a single-0 wheel used at the Barona Resort, but all of California requires the use of something in addition to the wheel/ball. Many times, a deck of cards is used to simulate the wheel/ball.

When the ball stops spinning, it will land in one numbered slot. If the winning slot matches your bet, you win money **and** get back the amount you bet too ☺! If not, the house takes your bet and keeps it. ☹

In Roulette, if you bet on something that is highly unlikely, like a single number, the payout goes way up. But if you bet on something that is pretty likely – like betting on red or black – the payout goes way down. In any game, we need a list of payouts (sometimes called payout odds) and the corresponding bet.

If the **payout odds** are 35:1, then the customer will win \$35 for every dollar bet. If you bet \$2, then you'll win \$70. Since you get your bet back after winning, you would walk away with \$72. If you bet \$10, you'll win \$350 but walkaway with \$360.

EXPLORE (1)! Determine the amount you win and the amount you could walk away with if the payout odds were…

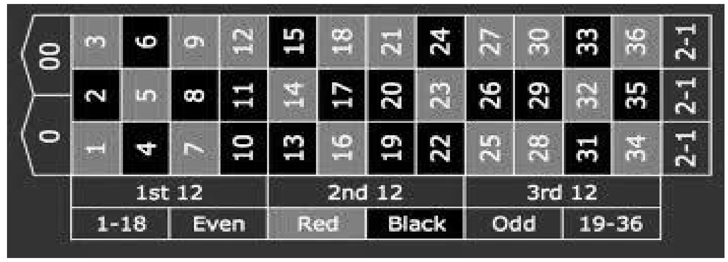
A) 2:1, and you bet \$10

B) 2:1, and you bet \$275

C) 5:1, and you bet \$60

D) 17:1, and you bet \$700

Here's a picture of the betting table for roulette and the list of payout odds that are typical at most casinos. Be aware, all payout odds are controlled by the casino and may not be the same from one place to another.



Number of ways to win	Bet Name	Bet Description	Payout Odds
1	Straight Up (single)	You put your chip(s) on one number	35:1
2	Split Bet (double)	Pick 2 touching numbers and put chip(s) on the line between them.	17:1
3	Street/Trio/Line Bet	You put your chip(s) on a line of 3 numbers, like 1-2-3 or 13-14-15.	11:1
4	Square/Corner/ Quad Bet	You put your chip(s) on the point where 4 numbers come together like 1-2-4-5 or 20-21-23-24.	8:1
5	Basket/ 5-number Line Bet	You bet on the 5 number grouping: 0-00-1-2-3 only.	6:1
6	Double Street/ 6-number Line Bet	You wager on 6 numbers on two consecutive streets like 1-2-3-4-5-6 or 10-11-12-13-14-15.	5:1
12	Column/Dozens Bet	Select one of the columns of 12 numbers, the long strips. 0 and 00 don't count for this bet. This is also for grouped dozens like 1-12.	2:1
18	High/Low Even/Odd Red/Black	Each of these bets will cover 18 numbers. Betting on High (19 – 36) or Low (1 – 18). NOTE: 0 and 00 are not included in any of these bets!	1:1

EXPLORE (2)! Go to https://casino.bovada.lv/table-games A list of games will pop up and we want the one in the lower left "American Roulette." When you hover over the top, click on the play now button, then choose "Practice Play." If you accidentally click on the "Real Play" it will be for actual money.

A) Click the chips to bet $25, then your teacher will assign you a bet. It's totally up to you which bet you make as long as it is the type stated. Each row in class will have a different type of bet. You must keep it consistent as you will make 38 bets in a row. Each person starts with $1,000 and in the space below, you'll keep track of how many bets you have made.

Make a tally mark | for each bet – 38 total		Ending balance after 38 bets
Bets where you lost	Bets where you won	

B) Take the amount of money you won (or lost) total and divide by 38. How much did you win (or lose) per bet on average?

EXPLORE (3)! After everyone is done, we can calculate for an entire class, the amount of money won or lost, and the average amount per bet on average.

A) What was this amount for the class?

B) Did you think that you would have done better if you had changed your bet?

C) Which bet seemed to do the best overall by having more wins?

D) Which bet seemed to do the worst overall by having more losses?

E) Looking at the payout odds table, about how many times (out of the 38 bets) would you expect to win if you bet on:

a. Straight up?
c. Basket bet?
e. Red?
b. Split bet?
d. Column bet?
f. Trio bet?

F) Explain why people would want to bet on a single number bet?

G) Explain why people would want to bet on red?

H) Explain the dilemma with Roulette when it comes to betting and payoffs.

<u>3.2: Introduction to Probability</u>

The key reason to learn the rules of a game like Roulette is to see a connection between rules of a game and the rules in math. Knowing the rules and having a strategy can increase your payout (or your grade).

<u>EXPLORE (1)!</u> Follow-up questions on Roulette.

A) While playing the game, did it seem easier to win on a 2-number bet or a 6-number bet?

B) While playing the game, did it seem easier to win on a column bet or a bet on red?

Comparing these two, it seems pretty clear that the 6-number bet is easier to obtain than the 2-number bet. In fact, there are 3 times more options when you compare 6 to 2. One very rough idea of probability is to think of the number of options for what you want out of the total. Let's formalize this idea.

All of these results depend on the **<u>probability</u>** or chance of the bet that you picked being the winner. When we consider these topics, some new terms are needed. Mathematically, we are talking about ways to **<u>quantify</u>** the idea of probability or to turn this new concept into numeric quantities.

We create experiments or **<u>procedures</u>** and then observe what happens. The result of a procedure is called an **<u>outcome</u>**. A group of possible outcomes makes up an **<u>event</u>**, and any event having exactly one outcome is known as a **<u>simple event</u>**. From Roulette, the procedure is putting the ball in and spinning the wheel, and the outcome is the result of the spin. Some events will have multiple outcomes (like a basket bet) while others have only one outcome (single number bet).

The **<u>sample space</u>** is the set (or collection) of all of the possible outcomes of the procedure.

While Roulette is fairly basic to picture all of the outcomes, a game like Yahtzee can also help us to see some of these concepts too; however, it is challenging to find the likelihood of individual events in Yahtzee game because of the incredible number of options when rolling 5 dice.

<u>NOTE:</u> Yahtzee is a dice game. When there is more than one, we use the word "dice." If there is only one being used, we use the word "die." So we would roll one *<u>die</u>*, but roll two *<u>dice</u>*. This may remind you of a great joke: How to keep your kids from gambling? You take away their paradise (pair-of-dice)!

Example: Write all the outcomes contained in the event from the procedure "roll one 8-sided die."

	Event	Actual Rolls (Outcomes)
A)	Roll an odd number	1, 3, 5, 7
B)	Roll a number greater than or equal to 5	5, 6, 7, 8
C)	Roll a number greater than or equal to 1	1, 2, 3, 4, 5, 6, 7, 8
D)	Roll a 7	7
E)	Roll an 8	8
F)	Roll a 0	Not possible
G)	Roll a number greater than 5	6, 7, 8

All of these are events, but only E and F are simple events. The sample space of this procedure is all of the simple events (outcomes): $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

EXPLORE (2)! Write all the outcomes contained in the event from the procedure "roll one 6-sided die." Write down all the outcomes that would make the following events occur.

	Event	Actual Rolls (Outcomes)	Simple Event?
A) **	Roll an odd number		Yes No
B)	Roll a number greater than or equal to 3		Yes No
C)	Roll a number less than or equal to 6		Yes No
D)	Roll an 8		Yes No
E)	Roll a 3		Yes No
F)	Roll a number greater than 3		Yes No

G) What is the sample space for the procedure listed above?

As we move through probability, we often use capital letters (A, B, C, etc) to refer to specific events. When we do this, it is important that the event is clearly defined so there is no confusion. Further, the probability of an event A is denoted $P(A)$ using a type of function notation. The input for this function is the event and the output is the probability (a number).

Examples:

- The probability of rolling a 3 in the previous exploration could be written as $P(E)$, or if the context is clear, we could write $P(3)$.
- The probability of rolling a number less than or equal to 6 could be written as $P(C)$, or if the context is clear, we could write $P(x \le 6)$ where x represents the number rolled on the die.

EXPLORE (3)! Try a few questions about outcomes.

A) Were there ever events where there were no possible outcomes? Give an example or explain why this can't happen. This type of event is called an **impossible event**.

B) Were there ever events where the possible outcomes were exactly the same as the sample space? Give an example or explain why this can't happen. This type of event is called a **certain event**.

When all of the outcomes of a procedure have the same chance of occurring, we call them **equally likely outcomes**. Most of what we will do involves equally likely outcomes, and this will lead us to general rules related to this specific type.

EXPLORE (4)! Determine which of these are equally likely outcomes.

	Outcomes	Equally likely?
A) **	Rolling a 1, 2, 3, 4, 5, or 6 on a 6-sided die.	Yes No
B)	Flipping a head or tail on a standard coin.	Yes No
C)	Drawing a Red, White, or Blue marble from a jar of 3 red, 2 white, and 1 blue marble.	Yes No
D)	Drawing a Red, White, or Blue marble from a jar with 10 red, 10 white, and 10 blue marbles.	Yes No

EXPLORE (5)! Find the sample space, event space, and the probability related to these procedures that produce equally likely outcomes.

	Procedure	Sample Space	Event	Probability
EX:	Flip a single coin, try to get H.	{H, T}	$E = \{H\}$	$P(E)=\frac{n(E)}{n(S)}=\frac{1}{2}$
A) **	Roll a 6-sided die; try to get a 3.			
B)	Roll a 6-sided die; try to get an even number.			
C)	Draw a marble from a jug of 4 red, 8 blue; try to get red.			
D)	Roll a 12-sided die; try to get a prime number.			

To help explore more, we need to create a way to measure probability. In mathematical terms, the formula we use will **quantify** the idea of chance. Start with some procedure that produces a sample space of equally likely outcomes, called *S*. Now form an event, *A*. The number of ways it could occur is $n(A)$ with the number of outcomes in the sample space of $n(S)$. Then the **probability** of event *A* is: $P(A)=\frac{n(A)}{n(S)}$.

When events are not equally likely, other techniques must be used to compute probability – but the concept here will remain constant: number of options out of number of the total. Notice that in the example above with marbles in a jug, there were 12 marbles. Drawing any one of those marbles would create equally likely outcomes. But the sample space showing {R, B} does not adequately reflect that, which is where we need to be very clear when we communicate with others.

It is important to clarify that each marble being drawn (an outcome) is equally likely, but each color being drawn (an event) is *not equally likely*.

EXPLORE (6)! Suppose we have a bag with 3 red marbles, 2 blue marbles, and 1 green marble. We will reach into the bag and randomly select one marble. Determine the probabilities of these events.

	Event Description	Probability
A) **	Draw a green marble.	
B)	Draw a blue marble.	
C)	Draw a red marble.	
D)	Draw a green *or* blue marble.	
E)	Draw a red *or* blue marble.	
F)	Draw a green *or* red *or* blue marble.	
G)	Draw a marble that is not blue.	
H)	Draw a marble that is blue *and* red.	

When we know the probability is some decimal or fraction, we could convert that to a percentage. So if $P(A)=0.57$, here are some ways to interpret it: (1) There is a 57% chance that the event A will occur or (2) The event A will happen 57 times out of 100 procedures (on average).

EXPLORE (7)! Circle the word that best explains the probability. The line represents a 50-50 chance.

	Probability	The event is…
A) **	$P(A)=0.57$	Impossible…Highly Unlikely…Unlikely. ¦ .Likely…Highly Likely…Certain
B)	$P(B)=0.17$	Impossible…Highly Unlikely…Unlikely. ¦ .Likely…Highly Likely…Certain
C)	$P(C)=0.97$	Impossible…Highly Unlikely…Unlikely. ¦ .Likely…Highly Likely…Certain
D)	$P(D)=1$	Impossible…Highly Unlikely…Unlikely. ¦ .Likely…Highly Likely…Certain
E)	$P(E)=\frac{498}{500}$	Impossible…Highly Unlikely…Unlikely. ¦ .Likely…Highly Likely…Certain
F)	$P(F)=\frac{8}{5{,}000}$	Impossible…Highly Unlikely…Unlikely. ¦ .Likely…Highly Likely…Certain
G)	$P(G)=\frac{1}{290{,}000{,}000}$	Impossible…Highly Unlikely…Unlikely. ¦ .Likely…Highly Likely…Certain

3.3: More Advanced Probability

As we progress with probability, the procedures often become more challenging. Instead of rolling one die, we could roll two dice. This section gives far more thought provoking examples.

Example (1): Determine all answers involving the procedure: "Flip a coin so the result is random and record the result."

A) What are the outcomes? Use the outcomes to create the sample space.
   a. The outcomes are H and T.
   b. The sample space would be $S = \{H, T\}$
B) Are the outcomes equally likely?
   a. These outcomes are equally likely (assuming it is a fair coin).
C) Determine probability of flipping heads and the probability of flipping tails.
   a. The events will be $A = \{H\}$ and $B = \{T\}$.
   b. $P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$ and $P(B) = \frac{n(B)}{n(S)} = \frac{1}{2}$

This gives us some techniques to find the probability for different procedures.

Interactive Example (2): Determine all answers involving the procedure: "Flip a coin twice and record the result in order."

A) What are the outcomes? Use this to create the sample space.

B) Are the outcomes equally likely? YES NO

C) Determine probability of …
   a. Flipping a coin twice and getting T-T.

   b. Flipping a coin twice and getting one H and one T.

EXPLORE (1)! Determine answers involving the procedure: "Flip a coin three times and record the result in order."

A) What are the outcomes? Use this to create the sample space.

B) Are the outcomes equally likely? YES NO

C) Determine probability of …
   a. Flipping a coin three times and getting T-H-T.

   b. Flipping a coin three times and getting two tails and one head.

   c. Flipping a coin three times and getting at least one head.

Interactive Example (3): Express the sample spaces from the previous problems to show connections.

1 coin flip 2 coin flips 3 coin flips

What pattern (or patterns) did you find?

EXPLORE (2)! Determine answers involving the procedure: "Flip a coin four times and record the result in order."

A) What are the outcomes? Use this to create the sample space – link to previous examples for a pattern.

B) Are the outcomes equally likely? YES NO

C) Determine probability of …

a. Flipping a coin four times and getting T-H-T-H.

b. Flipping a coin four times and getting two tails and two heads.

c. Flipping a coin four times and getting at least two heads.

d. Flipping a coin four times and getting no heads at all.

e. Flipping a coin four times and getting at least one head.

f. Is the probability of flipping a coin four times and getting H-H-H-T the same as flipping a coin four times and getting 3 heads and 1 tail? Why or why not?

g. How do these coin flipping results relate to Pascal's Triangle?

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Previously, we considered rolling a single die. Now we will roll two dice and see the results!

Interactive Example (4): Determine answers involving the procedure: “Roll two dice and record the result in order.”

A) What are the outcomes? Use this to create the sample space.

		Second Die					
		1	2	3	4	5	6
First Die	1	(1, 1)					
	2						
	3						
	4						
	5						
	6						

B) Are the outcomes equally likely? YES NO

C) How many total outcomes are there from this procedure?

D) Does this number match with what we would expect using our counting techniques? Explain.

Many games use multiple dice and then sums the numbers on the top after rolling them. Let’s see how that stacks up.

EXPLORE (3)! Determine the probabilities based on rolling two dice.

A) How many different ways can we roll two dice and get a sum of 3?

B) What is the probability that the sum of the faces is 3?

C) How many different ways can we roll two dice and get a sum of 5?

D) What is the probability that the sum of the faces is 5?

Interactive Example (5): Fill out the table below recording

A) the number of ways a sum can be rolled (using two dice).
B) the probability that a particular sum will be rolled (using two dice).

Sum is	**2****	**3**	**4**	**5**	**6**	**7**	**8****	**9**	**10**	**11**	**12**
Number of ways:	1										
Probability of the sum:	$\frac{1}{36}$										

C) Is each sum equally likely? Explain.

D) Since each outcome is equally likely when rolling two dice, explain the difference between an outcome and a sum.

E) Which sum is most likely when rolling two dice?

EXPLORE (4)! Based on the table on this page, determine the following probabilities.

A) The probability of a sum that is prime.

B) The probability that the sum is even.

C) The probability that the sum is greater than 7.

D) The probability that the sum is greater than 7 and even.

E) The probability that the sum is greater than 7 or even.

EXPLORE (5)! The gender of babies can be recorded as B for boy and G for girl. The procedure is to record the gender of the next 3 babies born (in order). [For now, let's assume that a B or G is equally likely!]

A) What are the outcomes? Use this to create the sample space.

B) Are the outcomes equally likely? YES NO

C) Determine probability of …
   a. The next 3 babies being B-G-G.

   b. The next 3 babies having the same gender.

EXPLORE (6)! The LA Galaxy were equally likely to win or lose games. The procedure is to record the wins and losses for the next 3 games played by the Galaxy.

A) What are the outcomes? Use this to create the sample space.

B) Are the outcomes equally likely? YES NO

C) Determine probability of …
   a. The Galaxy winning their next 3 games.

   b. The Galaxy winning 2 and losing 1 in the next 3 games.

D) Is there a connection between the last two sample spaces? Explain.

<u>For Love of the Math:</u> *A very famous column by Marilyn Vos Savant (woman with highest IQ ever recorded), described a problem with boys and girls. The story is in 2 parts:*

1. *Couple A has two children, and the oldest is a boy. What is the probability both children are boys?*
   a. *Draw out the sample space, and cross off options where the first child is not a boy. What are you left with? Which of these options gives both boys? Determine the probability from here.*
2. *Couple B has two children, and at least one is a boy. What is the probability both children are boys?*
   a. *More challenging – explore if interested!* https://en.wikipedia.org/wiki/Boy_or_Girl_paradox

<u>EXPLORE (7)!</u> For this next part, we will 'mix' procedures: now flip a coin (record the result) and then roll a 6-sided die (record the result).

A) Write the sample space for this procedure.

B) What is the probability of the following events:

	Event	**Probability**
a.	$P(H, 2)$	
b.	$P(T, \text{even number})$	
c.	$P(H, 5)$	
d.	$P(4 \text{ on the die})$	
e.	$P(H \text{ on the coin})$	

<u>Example (6):</u> How many groups of two can you make with the numbers 1, 2, and 3 if repetition is not allowed? [For this procedure, the *<u>order does matter</u>* so the group (2,3) is a different pair than (3,2).]

A) List the sample space of the permutations of 2 elements.

(1, 2)	*(2, 1)*	*(3, 1)*
(1, 3)	*(2, 3)*	*(3, 2)*
Start with 1, list options.	*Then start with 2, list options.*	*Then start with 3, list options.*

B) What is the probability that the randomly chosen pair is (5,2)?

Since the ordered pair is not in the sample space, the probability is 0.

C) What is the probability that a randomly chosen pair is (2, 3)?

Since there is only one of these out of the six options, the probability is $\frac{1}{6}$.

Example (7): How many combinations of two elements can you make with the numbers 1, 2, 3, 4, and 5? [Remember that with combinations, order does not matter so (2,4) is the same as (4,2)]

Let's be organized here so we don't leave out any pairs.

Start with 1	Start with 2	Start with 3	Start with 4	Start with 5
(1, 2) (1, 3)	(2, 1) (2, 3)	(3, 1) (3, 2)	(4, 1) (4, 2)	(5, 1) (5, 2)
(1, 4) (1, 5)	(2, 4) (2, 5)	(3, 4) (3, 5)	(4, 3) (4, 5)	(5, 3) (5, 4)

Now remove those that are already included, and work from left to right.

Start with 1	Start with 2	Start with 3	Start with 4	Start with 5
(1, 2) (1, 3)	~~(2, 1)~~ (2, 3)	~~(3, 1)~~ ~~(3, 2)~~	~~(4, 1)~~ ~~(4, 2)~~	~~(5, 1)~~ ~~(5, 2)~~
(1, 4) (1, 5)	(2, 4) (2, 5)	(3, 4) (3, 5)	~~(4, 3)~~ (4, 5)	~~(5, 3)~~ ~~(5, 4)~~

The total is 4 + 3 + 2 + 1 = 10, which is the same as ${}_5C_2$.

EXPLORE (8)! Consider the numbers 1, 2, 3, 4, and 5.

A) How many combinations of two can you make with the numbers 1, 2, 3, 4, and 5?

B) What is the probability of randomly choosing one combination of two and getting (4, 3)?

C) How many permutations of two can you make with the numbers 1, 2, 3, 4, and 5?

D) What is the probability of randomly choosing one permutation of two and getting (4, 3)?

E) If you could repeat the numbers selected, but were still using 1, 2, 3, 4, and 5, how many different ordered arrangements of two could be made? (repetition is possible)

F) What is the probability of randomly choosing a two element arrangement and getting (4, 3)?

3.4: Probability and Counting

As we saw near the end of the last section, counting and probability are very closely connected. Without counting techniques, finding the values to use for probability becomes nearly impossible. Let's review the counting learned in Unit 2:

- **Fundamental Counting Principle**
  - If you have *m* choices for the first option and *n* choices for the next, then there are $m \times n$ ways to pick one of each. It can be extended to as many options as you want.
- **Combinations**
  - The number of ways we can choose and arrange r objects from n objects where the order of arrangement *does not matter* is ${}_nC_r$. Remember the following tips:
    - There are *n* different (unique) objects to choose from.
    - We select *r* of the objects (without replacement)
    - We consider arrangements of items to be the same: abc is the same arrangement as bac, bca, acb, cab, and cba. When using combinations, all of these are not considered different and we count all of these permutations together as only one combination.
  - The formula for combinations is ${}_nC_r = \dfrac{n!}{(n-r)! \times r!}$.

So how can we use these formulas to help with probability?

Example (1): Determine the probability of selecting the blue shirt and brown pair of shorts if there are 4 shirts (black, green, blue, and purple) and 5 pairs of shorts (light tan, khaki, dark tan, brown, and black).

Solution: First, use the fundamental counting principal to determine how many different ways an outfit can be selected. Since there are 4 shirts and 5 shorts, then there are 4 × 5 = 20 total outfits possible. Only one of those is blue shirt and brown shorts, so the probability is $\frac{1}{20}$.

Example (2): Determine the probability of selecting an outfit that includes the blue shirt if there are 4 shirts (black, green, blue, and purple) and 5 pairs of shorts (light tan, khaki, dark tan, brown, and black).

Solution: First, use the fundamental counting principal to determine how many different ways an outfit can be selected. Since there are 4 shirts and 5 shorts, then there are 4 × 5 = 20 total outfits possible. The blue shirt can be selected with each pair of shorts, which gives 5 outfits, so the probability is $\frac{5}{20} = \frac{1}{4}$.

EXPLORE (1)! Test your ability with the Fundamental Counting Principal.

A) ** If your family has 4 children, what is the probability all the kids are girls? (assume having a boy or girl is equally likely)

B) If a restaurant serves 2 salads, 4 entrees, and 5 deserts, how many different meals can it serve?

  a. If a person randomly picks out items while blindfolded, what is the probability they order the garden salad, stuffed chicken and the chocolate volcano cake?

  b. Using the same choosing method, find the probability a person orders a meal with the Chocolate Volcano Cake.

C) If you roll a 4-sided die and a 6-sided die, what is the probability that you roll both and get a sum of 2?

Example (3): In a group of 10 people, 3 people are selected for a cleaning job. What's the probability that they are the tallest 3 people in the group?

Solution: First, use combinations to determine how many different ways a group of 3 people can be selected. Since there are 10 people and 3 to select, and since the cleaning job doesn't have any preferences between members, then we know that the order of selection doesn't matter. ${}_{10}C_3 = 120$, so there are 120 different ways to select the group of three people. Only one group of 3 will contain the 3 tallest people, so the probability is $\frac{1}{120}$.

Combination problems often relate to more challenging concepts, so here's a chance to review a bit from Unit 2 with counting.

EXPLORE (2)! How many different ways are there to select…

A) ** 8 doughnuts from a box of 12 doughnuts.

B) 7 boys from a group of 15 boys.

C) 14 women from a group of 25 women.

Example (4): In a group of 15 people, there are 5 women and 10 men applying for a job, but only 3 people will get it. What is the probability that the group selected has…

A) Exactly 3 women.
B) No women.
C) Exactly 1 woman.
D) Exactly 2 women.

Solution: First, use combinations to determine how many different ways a group of 3 people can be selected. Since there are 15 people and 3 to select, and since the job doesn't have any preferences between members, then we know that the order of selection doesn't matter. ${}_{15}C_3 = 455$, so there are 455 different ways to select the group of three people.

A) To select exactly 3 women, we need to think of how to choose the 3 women from the group of 5 women. Since there are 5 women and 3 to select, and since the order of selection doesn't matter, we can use combinations again. ${}_5C_3 = 10$, so there are 10 different ways to select the group of two women, meaning the probability is $\frac{{}_5C_3}{{}_{15}C_3} = \frac{10}{455} = \frac{2}{91} \approx 0.02198$.
B) This is similar to the previous part, except now we select 3 men and 0 women. The probability is computed as $\frac{{}_{10}C_3}{{}_{15}C_3} = \frac{120}{455} = \frac{24}{91} \approx 0.26374$.
C) In this last case, we need to select exactly 1 woman, which also means we need 2 men. Start by selecting the women: there are ${}_5C_1 = 5$ ways to select the 1 woman because there are 5 women to pick from. Now select the men: there are ${}_{10}C_2 = 45$ ways to select the 2 men. For each of the 45 ways to select the men, there are 5 ways to select the women, making a grand total of $({}_{10}C_2)({}_5C_1) = 45 \cdot 5 = 225$ ways to select exactly 1 woman and 2 men. The probability is computed as $\frac{({}_{10}C_2)({}_5C_1)}{{}_{15}C_3} = \frac{45 \cdot 5}{455} = \frac{225}{455} = \frac{45}{91} \approx 0.4945$.
D) This one is similar to the previous part, but now selecting more women. The probability is computed as $\frac{({}_{10}C_1)({}_5C_2)}{{}_{15}C_3} = \frac{10 \cdot 10}{455} = \frac{100}{455} = \frac{20}{91} \approx 0.2198$.

If you were counting, we could organize all the possibilities and show that they do indeed add up to the 455 total ways to select 3 people.

A) All women: 10 ways.
B) All men: 120 ways.
C) Exactly 1 woman: 225 ways.
D) Exactly 2 women: 100 ways.

Add these together and you have 10 + 120 + 225 + 100 = 455. ☺

Breaking these problems into pieces is important so do your best to see the groups in the numerator, and remember that the fundamental counting principle requires us to multiply those options together.

EXPLORE (3)! Test your probability-ability with Combinations.

A) ** There are 15 applicants but you can only interview 4. If you randomly select the applicants, what is the probability they are the youngest 4? (this type of problem is used in age discrimination cases.)

B) There are 30 people in a Math 52 class and you need to select 8 for an interview process. If you randomly select, what is the probability that you pick the shortest 8 people?

C) There are 30 people in a Math 52 class (11 men and 19 women) and you need to select 8 for an interview process. If you randomly select, what is the probability that you pick…

a. **(L)** Exactly 8 men

b. **(R)** Exactly 5 men

c. **(L)** Exactly 4 men.

d. **(R)** Exactly 5 women.

D) You have a group of 17 applicants to fill 3 jobs, and 10 applicants are women. They are all equally qualified for the job, so the company you work for has you randomly select to avoid bias. What is the probability that you…

a. **(L)** select all men?

b. **(R)** select all women?

c. **(L)** select at least one woman?

d. **(R)** select at least one man?

EXPLORE (4)! Now we put them together:

A) A combination lock has 4 spindles with 10 numbers on each spindle. How many different combinations can be made for the lock?

B) You serve 3 different sodas (Orange, Root Beer, and Black Cherry) and 3 different sandwiches (ham, turkey, and cucumber). What is the probability Charles randomly chooses Black Cherry and Ham?

C) You roll two 4-sided dice. What is the probability you roll a sum of 7?

D) The code to access a building is a 4-digit sequence from a ten number pad. What is the maximum number of attempts you would have to try to get into the building if you didn't have the code? (you cannot repeat a number)

E) There are 19 cell phones and 5 are defective. You pick 4 cell phones out of the batch. What is the probability that you randomly selected:

a. **(L)** 4 defective cell phones

b. **(R)** 3 defective cell phones

c. **(L)** 1 defective cell phone

d. **(R)** 0 defective cell phones

3.5: Classical vs Empirical Probability (Relative Frequency)

Since probability is so theoretical, it is important to realize that when we actually perform experiments, the results may not match the probability exactly.

Classical probability relates to equally likely outcomes: $P(A)=\frac{n(A)}{n(S)}$. Further, classical probability allows a computation based on what "should" happen, not what actually does happen.

But this really does assume that we know exactly what the sample space is and the number of outcomes. If we don't know those numbers, we can still create an experiment and repeat it over and over again. When the results are recorded, we can record the **relative frequency** (sometimes called **empirical probability**) by taking the number of times our outcome showed up and dividing by the number of total attempts.

EXPLORE (1)! Take a probability pig and toss it as you would toss a die.

A) How many different ways can the pig land? Which of these landing types is the most likely? (use the blank page to the left to keep track)

B) Between your group (row), roll the pigs at least 30 times each and record the results. As a class, which pig location is most likely and what is the relative frequency?

EXPLORE (2)! Determine which experiments use classical probability and which use empirical.

	Description	Type
A) **	You want to find out how likely it is that it will rain on May 4, so you look back in the almanac and find out how many rainy days there have been out of all the recorded days.	Classical Empirical
B)	Matt wants to know the chances of randomly selecting a MiraCosta student who is 21 years old or younger, so he asks everyone in the cafeteria about their age.	Classical Empirical
C)	Matt wants to know the chances of randomly selecting a MiraCosta student who is 21 years or older, so he asks the registrar for information about the age of all students.	Classical Empirical
D)	Patty wants to know the chances of rolling a sum of 7 on two 8-sided dice, so she rolls the dice 200 times and counts how many times she gets a sum of 7.	Classical Empirical
E)	Patty wants to know the chances of rolling a sum of 7 on two 8-sided dice, so she creates a table of all outcomes and finds which have a sum of 7.	Classical Empirical

Interactive Example (1): Years ago, Honey Nut Cheerios had Batman toys. There were 4 models and Scott purchased 7 boxes. What is the probability that Scott will get a full set?

We will use the computer to randomize the boxes and toys. On Excel, we would use the "=randbetween()" function. Between you and your group, run this experiment at least 20 times per person.

Directions: In cell A1, type in "=randbetween(1,4)" to represent one of the 4 toys being randomly selected in a box of cereal. Copy this down so there are a total of 7 cells with this formula. In cell C1, write the word "YES" and type 0 in C2, then in D1 write the word "NO" and type 0 in D2. Now, look and see if each number is shown: 1, 2, 3 and 4. If so, add 1 under the word yes, and if not, then add 1 under the word no. Every time you type in the result, a new set of data will be created.

A) How many of your 20 trials gave Scott a full set of Batman toys? What would this mean as a probability? Is it likely? Explain.

B) As a class, what are the chances with all individual results combined? Once combined, the information is known as **aggregate** data. If you separate it out, it is known as **disaggregating**.

C) What type of probability is this and why?

EXPLORE (3)! If an experiment was to flip a single coin 4 times in a row, how many times would you expect to get heads?

Your instructor will load the following clip on the main computer: https://youtu.be/NbInZ5oJ0bc The clip is the opening scene of *Rosencrantz and Gildenstern are Dead* (1990). Gary Oldman plays Rosencrantz, who discovers many laws of math and physics during the movie. In this scene, he finds a coin on the ground and begins to flip it. Tim Roth, playing Gildenstern, sees what is happening and decides to take a look after some interesting results.

Do your best to count how many times Rosencrantz flips heads before Gildenstern decides to take a look at the coin:

Gildenstern gives a speech (starting around 2:40 of the clip) about how this occurrence is not surprising, yet is actually quite surprising: *what are your thoughts?*

Picture yourself walking up to someone flipping a coin, a coin that they told you was a fair coin but you haven't actually looked at the coin. How many heads would they need to get in a row before **you** started to question whether it was a fair coin or not? Circle your answer:

Number of heads in a row when you call "malarkey": 2 3 4 5 6 7 8 9

EXPLORE (4)! Let's use the computer to 'flip' a coin 200 times and determine the number of heads and tails.

Directions: In cell A1, type in "=randbetween(1,2)" to represent heads and tails. Copy this down so there are a total of 200 cells with this formula. In cell C1, write the word "Heads" and type 0 in C2, then in D1 write the word "Tails" and type "=200 – C2" in D2. Now, go back to C2 and type in "=countif(A1:A200,1)" This will count every head in the list. Put your cursor in cell C4 and type in the number of heads showing. Every time you type in the result, a new set of data will be created, so just type in the number of heads and hit enter to repeat.

A) Run the test at least 10 times and record the number of heads you saw. How many would you have expected to get out of 200 flips?

Trial	1	2	3	4	5	6	7	8	9	10
Number of Heads										

B) Scroll down your list when finished with part (A). Determine the longest consecutive string of heads or tails when flipping a coin 200 times. Is this more or less than what you thought when you would have called "malarkey" earlier?

C) Is this surprising based on your previous answers? Explain.

For Love of the Math: *Some of you may found the clip interesting and even funny. The movie is a portrayal of Hamlet from the point of view of two minor characters.* https://youtu.be/KchhSIVwMdY?list=PL171C7AAC0587125B *This is clip #1 of 12 that show the whole movie, and the movie has a number of the funny scenes – some related to society and others to science.*

- *Part 1: 1:33 – the coin flipping scene.*
- *Part 1: 8:24 – Rosencrantz invents the Big Mac, but can't fit it in his mouth.*
- *Part 4: 0:25 – Rosencrantz discovers Galileo's law of falling bodies; epic fail when he tries to show it to Gildenstern at 1:45.* ☺
- *Part 5: 6:11 – Rosencrantz discovers Newton's conservation of momentum; epic fail again when he attempts to generalize and show his friend at 6:40.*
- *Part 6: 4:39 – Rosencrantz discovers the basics of a steam engine, more explicit at 5:55.*
- *Part 7: 4:37 – Rosencrantz discovers displacement of water.*
- *Part 9: 3:09 – Rosencrantz builds a paper airplane, then a bi-plane at 6:35.*

EXPLORE (5)! Let's make a deal! There was a TV show that offered contestants the chance to play a game and have some options to pick a prize. Sometimes, the prize was really bad – like a donkey.

In this version of the game, there are 3 curtains to choose from with 2 donkey prizes and one new car. You (the contestant) pick one curtain – hoping it is a new car. The host opens one door that is a donkey, and then asks you if you want to change your mind. Should you keep the original choice, or would you switch?

A) Do you feel there is any difference between keeping your choice or switching? Which option would you choose (switch or stay)?

B) You need to have at least one partner for this activity. Take a deck of cards with 2 of one type and 1 of another (like two 3's for the donkeys and one 5 for the new car).
   - One person is the host and the other is the contestant.
   - The host shuffles the cards and lays them face down, remembering the location of the car.
   - The contestant picks a card, and the host turns over one of the donkey cards.
   - The host asks if they want to switch or stay. Contestant chooses, and host reveals the car.
   - Record the following: Switch (and win/lose) or Stay (and win/lose) in the following table.

	Stay **(L)**	Switch **(R)**	Totals
Win			
Lose			
Totals			

C) Is this classical or empirical probability? Explain.

D) Based on the results, would you rather stay or switch? Explain.

EXPLORE (6)! Let's make a deal part 2! In this case, there are 100 curtains with 99 donkeys and 1 brand new car. Once again you pick one door, but this time, 98 donkeys are shown to you. Would you stay or switch now?

A) What is the probability that you picked the correct door at the beginning?

B) Does opening the doors change the probability that you picked the car on your first choice?

C) If you commit to switching in this scenario, before anything happened, what would be your probability of winning?

<u>EXPLORE (7)!</u> Use the table to determine some probabilities. A group of people at a mall are asked questions about a new perfume – rating it on a scale of 1 (hated it) to 3 (liked it).

	1 (hated it)	**2 (meh)**	**3 (liked it)**	**Totals**
Men	21	15	67	
Women	30	50	93	
Totals				

A) In the table above, fill in the totals for each row and column. How many men and how many women were surveyed?

B) **(L)** What are the chances of randomly selecting a person who hated the perfume from all people surveyed?

C) **(L)** What are the chances of randomly selecting a person who hated the perfume from all men surveyed?

D) **(R)** What are the chances of randomly selecting a person who liked the perfume from all women surveyed?

E) **(R)** What are the chances of randomly selecting a woman from all people who liked the perfume?

F) Is this classical or empirical probability? Explain.

3.6: Some Rules in Probability

With probability, we've already seen some things that are always true. This section brings what we've learned from previous sections and expands them to see the connections.

EXPLORE (1)! Answer some basic probability questions.
If C is an impossible event and D is a certain event, find…

A) $P(C)$

B) $P(D)$

C) Based on the previous parts, what are the smallest and largest values for a probability?

Example (1): Let's say that there is an event E where $P(E) = 0.35$. What is $P(\overline{E})$?

Event E has a 35% chance of happening, which means there must be 65% left that it won't happen. So $P(\overline{E}) = 0.65$.

EXPLORE (2)! Determine the missing probability.

A) ** If $P(G) = 0.24$, find $P(\overline{G})$.

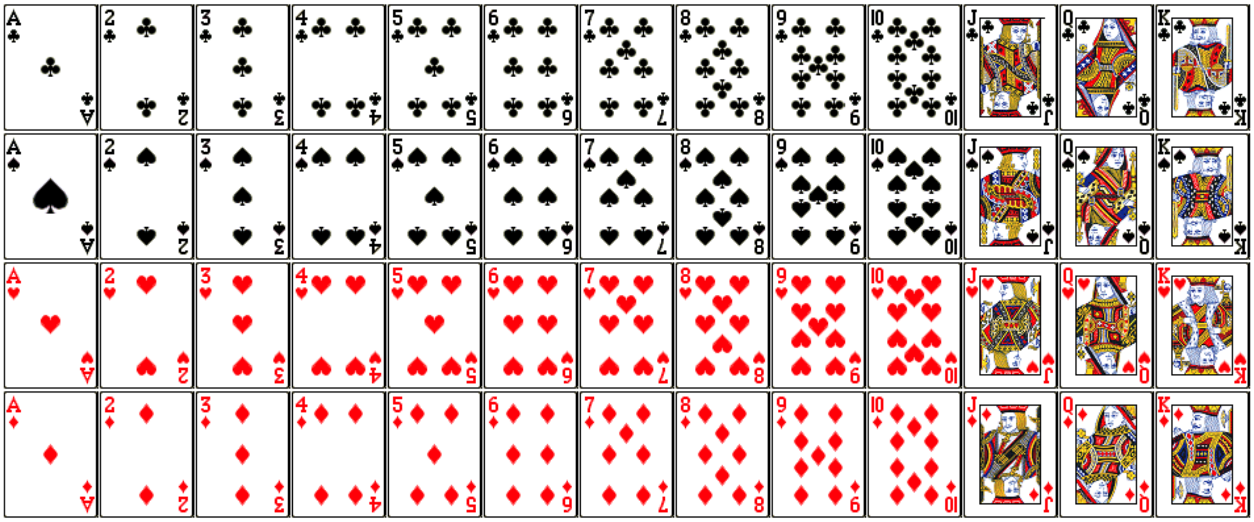
B) If $P(F) = \frac{3}{11}$, find $P(\overline{F})$.

C) If $P(\overline{H}) = 0.07$, find $P(H)$.

D) Event J is the event that you will roll a 6-sided die and get a 4. Find $P(\overline{J})$.

E) Event M is the event that you will flip a single coin twice and get T-H. Find $P(M)$ and $P(\overline{M})$.

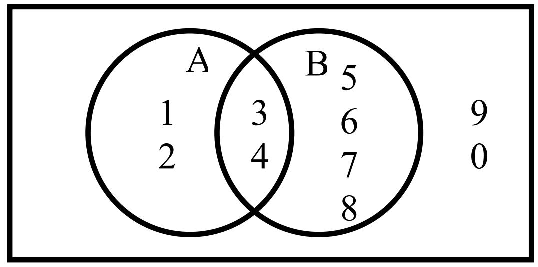
Now for some applications involving cards. A standard deck of cards has 52 cards; the cards are broken down into 4 suits (clubs – ♣, spades – ♠, hearts – ♥, and diamonds – ♦) and face values (Ace, 2. 3. 4. 5. 6. 7. 8. 9. 10. Jack, Queen, King). Here's a picture of all the card types. Your copy may show all as black, but the standard deck has clubs and spades being black with hearts and diamonds being red.



EXPLORE (3)! Determine the probabilities given below for a randomly drawn card.

	Description/Event *E*	$P(E)$	$P(\overline{E})$
A) **	Draw a 4.		
B) **	Draw a 4 or a 6.		
C)	Draw a spade.		
D)	Draw a spade and a 4.		
E)	Draw a heart or a club.		
F)	Draw a heart and a club.		
G)	Draw a heart and an ace.		
H)	Draw a club or spade or diamond.		
I)	Draw a card that is not a heart		
J)	Draw a card that is not an ace.		

EXPLORE (4)! In the following Venn diagram, the outcomes of a spinner are listed in different events. Find the probability of the given events.



A) $P(A)$

B) $P(B)$

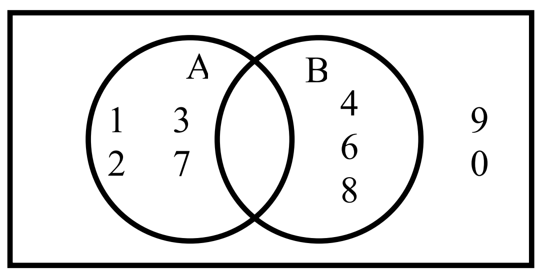
C) $P(\overline{B})$

D) $P(A \cup B)$

E) $P(A \cap B)$

F) Is $P(A \cup B) = P(A) + P(B)$ true or false here? Why?

EXPLORE (5)! In the following Venn diagram, the outcomes of a spinner are listed in different events. Find the probability of the given events.



A) $P(A)$

B) $P(B)$

C) $P(\overline{B})$

D) $P(A \cup B)$

E) $P(A \cap B)$

F) Is $P(A \cup B) = P(A) + P(B)$ true or false here? Why?

Recall from Unit 2, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. This rule can be expanded to include probability.

While it is always true that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for any two events A and B, is it ever true that $P(A \cup B) = P(A) + P(B)$? Explain **when** this is true.

Disjoint events occur in many different areas, so it is helpful to be able to determine whether events are disjoint or not. Remember that disjoint events cannot occur at the same time.

EXPLORE (6)! Determine if the two events are disjoint or not. Each should have a reason.

	Events	Conclusion
A)	Randomly choosing a person who was President of the USA. Randomly choosing a person who is a woman.	Disjoint Not Disjoint
B)	Randomly choosing a person who was President of the USA. Randomly choosing a person who is African-American.	Disjoint Not Disjoint
C)	Randomly choosing a high school student. Randomly choosing a volleyball player.	Disjoint Not Disjoint
D)	Randomly choosing a football player. Randomly choosing a female high school student.	Disjoint Not Disjoint
E)	Randomly selecting a pair of sunglasses. Randomly choosing an X-box game.	Disjoint Not Disjoint
F)	Randomly choosing a pair of prescription glasses. Randomly choosing a pair of sunglasses.	Disjoint Not Disjoint
G)	Randomly selecting a car. Randomly selecting a motorcycle.	Disjoint Not Disjoint
H)	Randomly choosing someone who smokes. Randomly choosing someone that doesn't have cancer.	Disjoint Not Disjoint

EXPLORE (7)! In 2010 the MiraCosta College student body had the following breakdown by race:

	Hispanic	White	Black	American Indian	Asian & Pacific Islander	Other	Totals
Population	91,120	262,244	13,977	1,577	32,421	13,646	414,985

A) Are the categories "White" and "Hispanic" mutually exclusive (disjoint) in the table above? Why?

B) If you randomly selected a person from the district what is the probability that the person is either Hispanic or White?

Summary of Current Probability Rules:

RULE #1: $0 \le P(E) \le 1$ for any event *E*; $P(E) = 0$ when *E* is an impossible event and $P(E) = 1$ when *E* is a certain event.

RULE #2: $P(E) + P(\overline{E}) = 1$ for any event *E*. This can be written in many ways: $P(\overline{E}) = 1 - P(E)$ or $P(E) = 1 - P(\overline{E})$ are equivalent. $\overline{E}$ and E are called **complementary events**.

RULE #3: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for any events *A* and *B*.

RULE #4: $P(A \cup B) = P(A) + P(B)$ for any disjoint (mutually exclusive) events *A* and *B*.

PROBABILITY IN ACTION

Interactive Example (2): When breathalyzer tests are examined, the result could be to test positive (+) which would mean that the test indicates the person is legally drunk, or test negative (–) which means the test indicates the person is not legally drunk. However, because of differences in human biology and chemistry, it is possible that the breathalyzer is incorrect – a person who is not legally drunk may test positive. Here's a table of results from breathalyzer tests.

	Is Legally Drunk	**Is Not Legally Drunk**	**Totals**
Test is Positive	103	22	
Test is Negative	5	167	
Totals			

A) Find the 'totals' for each row and column. How many total people are in this sample?

B) If LD is the event that someone is actually legally drunk, find $P(LD)$.

C) If Pos is the event that someone tests positive, find $P(Pos)$.

D) Find $P(Pos \cap LD)$ from the table and interpret what it means.

E) Find $P(Pos \cup LD)$ from the table and interpret what it means.

F) Is $P(Pos \cup LD) = P(Pos) + P(LD)$? Explain why or why not.

G) Find $P(Neg \cup \overline{LD})$.

H) If we put all of the people who tested positive in a group (ignoring all others), what is $P(LD)$ now?

I) Explain why $P(LD)$ is different in the two parts of this question.

<u>3.7: Conditional Probability, and Independent Events</u>

When we first talked about disjoint events, we saw that disjoint events would never happen at the same time. There are other types of events that can happen together, but knowing that one event occurred will change the probability of another event. If the result of one event influences the probability of the second event, we say the two events are **<u>dependent</u>**. If the result of the first doesn't influence the probability of the second, we say that the two events are **<u>independent</u>**.

<u>Examples of dependent events (1):</u>

A) Being dealt an Ace on the first card in blackjack; Being dealt an ace on the second card in blackjack.
B) Flipping a coin and getting H; Flipping the coin again and getting T.
C) Drawing 3 green skittles from one bag; Drawing a 4^{th} skittle and it is also green.
D) Sky is cloudy; It is raining.

<u>Examples of independent events (2):</u>

A) Flipping a coin and getting H; Drawing a card from a standard deck and getting a Jack.
B) Rolling a 7 on the Roulette wheel and then rolling a 23 on the Roulette wheel.
C) Drawing 3 Skittles from a bag and getting all green; Rolling a die and getting a 2.
D) It is sunny; I get a piece of junk mail delivered.

Because it may be hard to determine whether two events are dependent or independent, mathematicians came up with a way quantify the test – to turn it into numbers. This involves a new concept of **<u>conditional probability</u>**. In conditional probability, additional information is given to you and you need to decide what happens if we know the new information. This is very similar to conditional statements in Unit 1. The notation for conditional probability is this: $P(A|B)$, and it is read 'probability of *A* given *B*.' Another way to think of this is as follows: If you know that event *B* has happened, what's the probability of event *A* now?

<u>Example (3):</u> Let's say that our car was hit by someone driving a blue Prius who drove away. Would you have a better chance of finding the blue Prius if you knew that the car had Colorado plates? YES! So the probability of finding the car that hit you would change based on this additional event.

In the last section, we saw a table about a breathalyzer and someone being legally drunk. Let's test to see if the events "Testing Positive" and "Is Legally Drunk" are independent, so we'll need to find conditional probability. This interactive example will show us how to compute conditional probability.

<u>Interactive Example (4):</u>

	Is Legally Drunk	**Is Not Legally Drunk**	**Totals**
Test is Positive	103	22	125
Test is Negative	5	167	172
Totals	108	189	297

A) Find $P(Pos)$.

B) Find $P(Pos \mid LD)$ using this: If you know that the person is Legally Drunk, recalculate $P(Pos)$. Based on that, the formula for conditional probability is this: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{n(A \cap B)}{n(B)}$. This could be re-written for future use as $P(A \cap B) = P(B) \cdot P(A \mid B)$.

	Is Legally Drunk	**Is Not Legally Drunk**	**Totals**
Test is Positive	103	22	125
Test is Negative	5	167	172
Totals	108	189	297

EXPLORE (1)! Find the probabilities based on the table. Cover up options with conditional probability.

A) $P(LD \mid Pos)$

C) $P(Neg \mid \overline{LD})$

B) $P(LD)$

D) $P(Neg)$

E) Is it true that $P(LD) = P(LD \mid Pos)$? Explain what this means.

To compute whether two events are independent or not, we'll use conditional probability. If *A* and *B* are independent, then the events don't affect each other, so $P(A \mid B) = P(A)$.

This "test" for independence can be re-written using a little algebra into something that is equivalent:

$$P(A \mid B) = P(A) \Rightarrow \frac{P(A \cap B)}{P(B)} = P(A) \Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

After this extra work, we can clean up our definition. Events *A* and *B* are **independent** if and only if $P(A \cap B) = P(A) \cdot P(B)$. This is a modification of the general formula $P(A \cap B) = P(B) \cdot P(A \mid B)$ from the top of the page where we substitute $P(A \mid B) = P(A)$.

EXPLORE (2)! Based on your experience, determine if the following pairs of events are independent or dependent.

	Events	**Conclusion**
A)	Your car radio doesn't work. Your car doesn't start.	Independent Dependent
B)	Your cell phone doesn't work. Your car doesn't start.	Independent Dependent
C)	You are in a math class. You are carrying a cell phone.	Independent Dependent
D)	You are in a math class. You are carrying a calculator.	Independent Dependent

EXPLORE (3)! Determine the answers based on this frequency table about political candidates:

	Ted Cruz (*TC*)	**Donald Trump (*DT*)**	**Marco Rubio (*MR*)**	**Other/Undecided (*OU*)**	**Totals**
Male (*M*)	45	93	56	11	
Female (*F*)	209	116	257	33	
Totals					

A) Find $P(F)$.

B) Find $P(DT)$.

C) Find $P(F \cap DT)$.

D) Find $P(F \mid DT)$.

E) Are the events F and DT independent? Explain.

F) Find $P(M \cap MR)$.

G) Find $P(TC \cap DT)$.

H) Find the probability that a randomly selected person was Undecided.

I) Find the probability that a randomly selected person was Undecided given that the person was a male.

J) Based on the previous answer, are the events "Undecided" and "Male" independent? Explain.

K) If $P(A \mid B) = 0.57$ and $P(A) = 0.53$, are events A and B independent? Why or why not.

L) If $P(A \mid B) = 0.57$ and $P(A) = 0.57$, are events A and B independent? Why or why not.

3.8: Multiplication Rule, Complementary Events and "At Least 1"

What if you accidentally went to the wrong class one day? You showed up, sat down and then you were given a two question quiz that you had no idea about.

1. Ce que votre chien mange des carottes ? Yes No
2. What is the first letter on page 27 of the book you were supposed to read last night ?
   a. b. c. d. e.

Of course, this would be a pretty bad situation. You'll have to guess, so circle your options above.

Now, let's try to figure out your chances of getting both questions correct by listing out the sample space.

S = {YA, YB, YC, YD, YE, NA, NB, NC, ND, NE}

Since only one of these outcomes is correct, our probability of getting it correct is $\frac{1}{10}$.

EXPLORE (1)! Determine the probabilities listed.

A) What is the probability of guessing and getting the first question correct in the quiz above?

B) What is the probability of guessing and getting the second question correct in the quiz above?

C) What happens when you multiply the results you just obtained? (keep them as fractions)

D) How well does your result match the $\frac{1}{10}$ obtained previously? Coincidence?

In the previous section, we saw that $P(A \cap B) = P(B) \cdot P(A \mid B)$ for all events A and B. But when the events were independent, we saw a slightly different formula: $P(A \cap B) = P(A) \cdot P(B)$.

In our first example, with the two-question quiz, the events are independent because knowing the answer to the first answer doesn't change the probability of getting the second answer correct.

The next few rules are sometimes called the Multiplication Rules of Probability:

RULE #5: $P(A \cap B) = P(B) \cdot P(A \mid B)$ for any events A and B.
RULE #6: $P(A \cap B) = P(A) \cdot P(B)$ for any *independent* events A and B.

Example (1): You have to randomly choose 2 numbers from: 1, 2, 3, 4, 5. After choosing the first number, replace it into the set and choose a second number. Repetition is allowed – so you could draw (2, 4) as well as (2,2). Find the probability of choosing (3, 5).

Solution: Since we are replacing the number after the first choice, the probability of the second choice won't be affected by our first choice, so our individual draws are *independent events*, and we can use $P(A \cap B) = P(A) \cdot P(B)$. The probability of picking a 3 first is $\frac{1}{5}$, and the probability of picking a 5 second is $\frac{1}{5}$ as well. $P(A \cap B) = P(A) \cdot P(B) = \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25}$.

Example (2): You have to randomly choose 2 numbers from: 1, 2, 3, 4, 5. After choosing the first number, you do not replace it, so your second choice only has 4 options. Find the probability of choosing (3, 5).

Solution: Because we don't replace the first number, the probability of drawing a 5 second will change based on our first draw. The probability that we will draw a 5 is different than the probability that we'll draw a 5 after drawing the 3 first. We will need to use RULE #5: $P(A \cap B) = P(B) \cdot P(A \mid B)$.

The probability of drawing a 3 first is still $\frac{1}{5}$, but the probability of picking a 5 second (after drawing the 3 first) is now $\frac{1}{4}$. So $P(A \cap B) = P(B) \cdot P(A \mid B) = \frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20}$.

EXPLORE (2)! What is the probability that you will draw from a deck of cards…

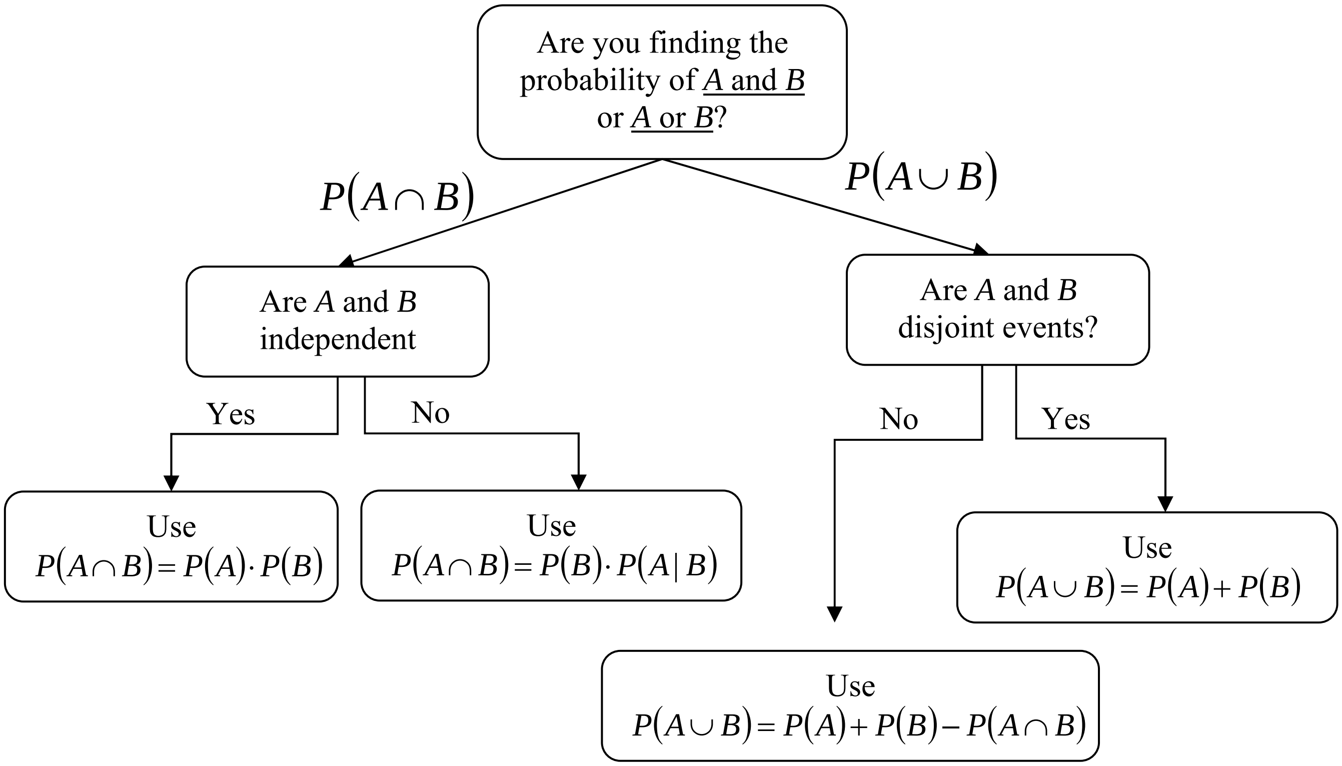
A) 2 hearts in a row with replacement?

B) 2 hearts in a row without replacement?

C) 2 aces in a row with replacement?

D) 2 aces in a row without replacement?

Here's a flow-chart image that can help:



EXPLORE (3)! Use your knowledge (and the flowchart above if necessary), to determine the following:

A) In order to win a game on your next turn, you need to roll a 6-sided die twice with a 2 on the first roll and a 6 on the second roll. What is the probability you win the game on your next turn?

B) You have six ping pong balls in a bag numbered 1 thru 6. Two balls are drawn, what is the probability that you randomly draw 6-5…

a. without replacement.

b. with replacement.

C) What is the probability that you will win on two single number bets in a row in Roulette?

D) Yazmin bought 20 flashlights. She knows from the manufacturer that there is a 90% chance each flashlight will work – it is only 90% because of the bad batteries they may be shipped with. If you randomly choose flashlights, what's the probability …

a. you choose 2 flashlights & both work?

b. you choose 5 flashlights and all work?

With these new formulas for probabilities, we can solve many problems. For example, we could solve a problem like this next one that involve "**at least one**" situations:

Example (3): Find the probability that you reach into a bag with 10 red marbles and 15 blue marbles and draw at least one red marble in the first 3 draws without replacement?

Solution: There are 4 possibilities here; we draw exactly 3 red marbles, exactly 2 red marbles, exactly 1 red marble, or exactly 0 red marbles. Since the directions say "at least one" red marble, then there are 3 cases we need to deal with: exactly 3 red marbles, exactly 2 red marbles, and exactly 1 red marble. Now, in order to compute these probabilities, we need to do all 3 of these.

But, since all probabilities must add up to 1, we could just find the one time it fails and subtract from 1.

Probability of at least one red marble =

1 – Probability of no red marbles = $1-\frac{{}_{15}C_3}{{}_{25}C_3}=1-\frac{455}{2300}=\frac{369}{460}=0.8022$.

Example (4): Find the probability that you roll a single 6-sided die three times and got *at least one* 6?

Solution (**METHOD 1** – Brute force): We will need to find all possible outcomes and determine the results. To determine the possibilities, we'll say that S represents success (rolling a 6) and N represents non-success (rolling anything else). Then our sample space is: *SSS*, *SSN*, *SNS*, *NSS*, *NNS*, *NSN*, *SNN*, *NNN*. However, these are not equally likely outcomes, so we'll need to compute each one. Since they are all different, to find the probability we can just add the individual probabilities up using an extension of $P(A\cup B)=P(A)+P(B)$.

- $P(SSS)=\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{216}$
- $P(SSN)=\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}=\frac{5}{216}$
- $P(SNS)=\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}=\frac{5}{216}$
- $P(NSS)=\frac{5}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}=\frac{5}{216}$
- $P(NNS)=\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}=\frac{25}{216}$
- $P(NSN)=\frac{5}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}=\frac{25}{216}$
- $P(SNN)=\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}=\frac{25}{216}$
- $P(NNN)=\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}=\frac{125}{216}$

So the probability of getting at least one 6 is:

$P(SSS)+P(NSS)+P(SNS)+P(SSN)+P(NNS)+P(NSN)+P(SNN)=$

$\frac{1}{216}+\frac{5}{216}+\frac{5}{216}+\frac{5}{216}+\frac{25}{216}+\frac{25}{216}+\frac{25}{216}=\frac{91}{216}$. Phew – that does it.

Looking back, this was an enormous pain in the butt. Finding 7 different probabilities and adding them up was time consuming. Perhaps we can use something else from this section and make this process easier.

In this case, event *E* will represent rolling at least one 6 in three rolls of a 6-sided die. That means $\overline{E}$ will be the event that you don't roll at least one 6, and from the chart above, the only way that happens is that we roll no 6's at all which is only one case – *NNN*! Recall our complementary events: $P(E)+P(\overline{E})=1$ for any event *E*. For what we will do, it may be helpful to rewrite this as $P(E)=1-P(\overline{E})$.

<u>Solution (**METHOD 2** – Complementary Events):</u> We will need to find all possible outcomes and determine the results. To determine the possibilities, we'll say that *S* represents success (rolling a 6) and *N* represents non-success

So to find the probability, we really only need $P(\overline{E})=P(NNN)=\frac{125}{216}$. So $P(E)=1-\frac{125}{216}=\frac{91}{216}$.

It is up to you to determine which of these methods will work best for a specific problem, and there are some opportunities on the next few pages to determine an efficient method.

<u>**EXPLORE (4)!**</u> Solve the following probability problems using the method of your choice.

A) If there is a 13% chance of drawing 8 cards in a row and getting no spades, what is the probability that you draw 8 cards in a row and get at least one spade?

B) If there is a 2.5% chance of rolling 6 die and getting no fives, what is the probability that you roll 6 die and get at least one five?

C) If the probability of getting at least one winner in 10 games is 87.4%, what is the probability of getting no winner in 10 games?

EXPLORE (5)! Solve the following probability problems using the method of your choice.

A) A dice game is made with a group of people. The first person (Roller) rolls an 8-sided die and that number is written down. The rest of the people (Group) have one roll each to match this number. If any of the Group rolls the same number as the Loner, the Group wins. What is the probability the Group wins if there are 4 people in the group?

B) Repeat the game from above, but this time use 10 people in the group.

C) ** In a group of 20 people, what is the probability that at least one of them shares your birthday? (For this, let's assume there are 365 birthdays possible.)

D) ** In a group of 20 people, what is the probability that at least two people in the group share a birthday?

E) What is the difference between parts (C) and (D)?

F) In a group of 35 people, what is the probability that at least 2 people in the group share a birthday?

<u>EXPLORE (6)!</u> With the number of techniques we've used in this unit, a few questions to bring them all together may be helpful. For this group we will be drawing numbers out of a bag containing the ten numbers from 1 to 10… find the following:

A) The probability of drawing a 6.

B) The probability of not drawing a 6.

C) The probability of drawing a 6 or a 3.

D) The probability of drawing a number that is an even number or is greater than 5.

E) Probability of drawing a 6, then drawing a 4 with replacement.

F) Probability of drawing a 6, then drawing a 4 without replacement.

G) Probabilities of having 5 people draw a number with replacement and at least one draws a 6.

H) The probability of drawing two numbers with replacement and have them be a 6 and a 3 in that order.

I) The probability of drawing two numbers without replacement and have them be a 6 then a 3, in that order.

J) The probability of drawing two numbers without replacement and have them be a 6 and a 3, in <u>any</u> order.

K) The probability of drawing four numbers without replacement and ending with 5, 8, 2, and 5, in that order.

3.9: Odds and Probability

We saw payout odds in the first section with Roulette. Those are related to how much money we get paid when we win. While probability is one way to report chances of winning, another technique is also used at times. For any event with a probability, the odds are related closely to the probability as described here:

Odds in favor of an event E are $O(E) = n(E) : n(\overline{E})$, which is one way of saying that the ratio is 'number of successes' to 'number of failures.' Often, in casinos or for sports betting, the odds are reported the opposite way; the **odds against** an event E happening are $O(\overline{E}) = n(\overline{E}) : n(E)$.

Example: If $P(E) = \frac{3}{4}$, find $O(E)$ and $O(\overline{E})$.

Solution: The probability indicates that the event E happens 3 times for every 4 experiments, which means that it happens 3 times for every 1 time it doesn't happen. So $O(E) = 3:1$, which means $O(\overline{E}) = 1:3$.

EXPLORE (1)! Fill out the rest of the table.

	$P(E)$	$O(E)$	$O(\overline{E})$
A) **	0.2		
B)		5:7	
C)			13:1
D)	$\frac{15}{801}$		
E)		68:9	
F)			3:5

EXPLORE (2)! Connecting odds and probability.

In the 2015 Belmont Stakes, the odds listed for some horses at post time are given. When odds are stated in gambling, they are listed as the *odds against*. Determine the probability that the horses will win:

A) ** Tale of Verve (15:1)   B) Frammento (30:1)   C) American Pharoah (3:5)

NOTE: It is important to clarify that *odds* relate to probability, or likelihood, while *payout odds* relate to the amount of money paid out in a specific situation.

3.10: Expected Value

Probability has many applications to our lives and some of them get very complex.

- Insurance companies use mathematicians called Actuaries to compute probabilities and risk of an insured person, which is how an insurance premium is calculated.
- Photocopiers use probability models related to "fuzzy" logic to determine the ideal mix of temperature, toner, and electric charge to make the best copies. It also can be used to determine that a jam has occurred, and that's why sometimes it says there is a jam but there really isn't one.
- Weather forecasters use advanced probability models to determine the likelihood of rain on a particular day. That's why they often say the chance of rain is 30% or 50%, but rarely say 100%.
- Stock market and investing uses a lot of probability. People invest in a company based on predictions the company makes about earnings and revenue. You can bet with the company or you can bet against them all depending on your probability calculations.
- Sporting events like horse racing, baseball, and football have a lot of probability associated with them. Sometimes a casino will post a probability and people will decide to bet with the house or against it. The folks who make up the probability (or odds) are using extremely complicated probability formulas, and often making a lot of money.
- Gambling and gaming. This can be as basic and fun as Yahtzee, Candyland, or Monopoly, and as intense and lucrative as poker, blackjack, and the lottery.

As this is an introductory course, we will focus on the area that requires the least amount of highly complex mathematical models but is still very useful!

Expected value is a mathematical way to **quantify** complicated events, with outcomes having different probabilities. Let's consider Roulette and look at a 2-number bet which has payout odds of 17:1.

Example (1): We'll organize our thoughts in a table for a $1 bet.

Event	Number of outcomes	Value of each outcome	Total of the individual values
Win	2	17	34
Lose	36	– 1	– 36

Since you either win or lose on this bet, these events are mutually exclusive. So in order to find the total of all the individual values, we can just add them together. $34+(-36)=-2$. There were 38 total outcomes, so we can expect (on average) to end up with $\frac{-2}{38}\approx -0.05263$, or a loss of about 5.26 cents per $1 bet.

If you think of every possible outcome, and what the value of that outcome is, we could add them up and divide by the number of outcomes. The result, in one sense, is an "average" of what we could expect over the long run.

Example (2): Now take the same idea, and write it out mathematically:

$$EV=\frac{17(2)+(-1)(36)}{38}=\frac{17(2)}{38}+\frac{(-1)(36)}{38}=17\left(\frac{2}{38}\right)+(-1)\left(\frac{36}{38}\right)\approx -0.05263$$

This new method shows that we can represent expected value as the sum of (value)×(probability) for every outcome. You might find example 1 or example 2 easier to work with, so let's try a few more examples.

Example (3): Calculate the expected value of a $1 bet on "Black" in Roulette, with payout odds of 1:1.

Solution (first way):

Event	Number of outcomes	Value of each outcome	Total of the individual values
Win	18	1	18
Lose	20	– 1	– 20

Add the totals at the end: $18+(-20)=-2$; now divide by the total outcomes: $\frac{-2}{38}\approx -0.05263$, or a loss of about 5.26 cents per $1 bet.

Solution (second way):

In order to win, there are 18 ways out of 38, for a probability of $\frac{18}{38}$, and losing has a probability of $\frac{20}{38}$.

The value for winning is $1 and the value for losing is $ –1. So the expected value is:

$EV=1\left(\frac{18}{38}\right)+(-1)\left(\frac{20}{38}\right)=-\frac{2}{38}\approx -0.05263$, or a loss of about 5.26 cents per $1 bet.

Both methods get you the answer, but you may like one over another. If the expected value of an event is $0.25 on a $1 bet, then if you keep making $1 bets long enough you'll end up with an average of winning 25 cents on each bet. If the expected value is negative, like $ – 0.05263, then if you keep making $1 bets long enough you'll end up with an average of losing about 5 cents on each bet.

Example (4): Find the expected value of a game that costs $5 to play, and if you flip two coins and get heads on both, you'll win $19. Otherwise, you lose the cost you paid to play.

Solution: From this unit, we know the probability of flipping two coins and getting H-H is $\frac{1}{4}$, so the probability that you don't get H-H is $\frac{3}{4}$. The expected value is: $EV=(19)\left(\frac{1}{4}\right)+(-5)\left(\frac{3}{4}\right)=1$. This means that if you pay $5 to play this game a large number of times, you'll win $1 per game over the long run.

EXPLORE! Find the expected values for these realistic situations.

A) An insurance company has an auto insurance policy that costs $50 per month and will pay out $50,000 if you get in an accident. The probability of you getting into an accident is $\frac{1}{40{,}000}$.

B) An investment company offers an investment stock that requires a $50,000 investment. There is some risk and there is a 10% chance of losing $30,000, but if not then you will gain $20,000.

3.11: Expected Value and Gambling

Most casino games have an expected value that is negative, because they want to make a profit. However, without some good prizes that seem attainable, no one will play. Because most people come into the casino with money, we should be clear about the definition of how much money is ‘won’ compared to how much a person ‘walks away with.’ One of my friends went into a casino with $200 and came out with $50 telling me, “I won $50!”

When we discuss gambling, we need to know whether our money comes back in the course of the bet or whether the house keeps the bet.

- In a lottery or keno, we pay $1 and we don’t get our $1 back separately. Coming in with $1 and walking out with $100 means you won $99.
- In roulette or craps, if we pay $1 and win, then our winnings are put on the bet chip and passed back. If the winnings were $100, then you walk away with $101.

We’ll need to be very clear about the difference between winnings and the amount we walk away with as this will influence the expected value.

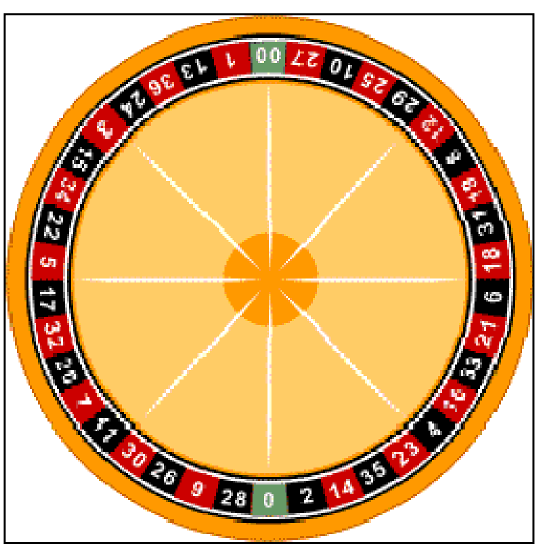
EXPLORE (1)! Determine the missing information.

A) If you bring $5 to a casino game and walk away with $20, how much did you win?

B) If you bring $5 to a casino game and win $50, how much do you walk away with?

C) If you bring $400 to a casino and leave with $200, how much did you win?

Roulette



An American roulette wheel has 18 black numbers, 18 red numbers and 2 green numbers (0 and 00). The 36 numbers that are black or red are split evenly between even and odd (1 – 36). The picture above shows the way the wheel is set up; Roulette is relatively easy to play as a player puts down chips of different values as a bet. The house spins the wheel and drops a ball in.

NOTE: There are other roulette wheels as well, and one version has only one green number – 0. This type is sometimes called European, but the 2-green number wheels are also used in Europe. There is a single-0 wheel used at the Barona Resort, but all of California requires the use of something in addition to the wheel/ball. Many times, a deck of cards is used to simulate the wheel/ball.

NOTE: The payout odds are not related to the probability. Odds of 1:1 mean the event happens 50% of the time, but payout odds of 1:1 mean you'll win $1 for every $1 you bet.

Betting table for roulette:

Number of winning options	Bet Name	Bet Description	Probability	Payout Odds
1	Straight Up (single)	You put your chip(s) on one number	$\frac{1}{38}$	35:1
2	Split Bet (double)	Pick 2 touching numbers and put chip(s) on the line between them.	$\frac{2}{38}$	17:1
3	Street/Trio/Line Bet	You put your chip(s) on a line of 3 numbers, like 1-2-3 or 13-14-15.	$\frac{3}{38}$	11:1
4	Square/Corner/ Quad Bet	You put your chip(s) on the point where 4 numbers come together. 1-2-4-5 or 20-21-23-24 are examples.	$\frac{4}{38}$	8:1
5	Basket/ 5-number Line Bet	You bet on the 5 number grouping that is 0-00-1-2-3 only.	$\frac{5}{38}$	6:1
6	Double Street/ 6-number Line Bet	You wager on 6 numbers on two consecutive streets. 1-2-3-4-5-6 or 10-11-12-13-14-15 are examples.	$\frac{6}{38}$	5:1
12	Column/Dozens Bet	Select one of the columns of 12 numbers, the long strips. 0 and 00 don't count for this bet. You can also get this payout for the grouped dozens like 1-12 or 13-24.	$\frac{12}{38}$	2:1
18	High/Low Even/Odd Red/Black	Each of these bets will cover 18 numbers. Betting on High (19 – 36) or Low (1 – 18). NOTE: 0 and 00 are not included in any of these bets!	$\frac{18}{38}$	1:1

EXPLORE (2)! Find the expected value of the following bets in roulette (do $1 bets only):

A) ** Single Number Bet

B) ** Square Bet

C) Split Bet

D) Column Bet

E) Red

F) Basket Bet

In this unit, it is easiest to compute $1 bets with the expected value. What this produces is an **expected value per dollar bet**.

Example (1): Find the expected value of …

A) a $1 bet in roulette on "High."
B) a $20 bet in roulette on "High."
C) a $35 bet in roulette on split bet (2-number bet).
D) a $57 bet in roulette on a square bet (4-number bet).

Solutions:

A) There are 18 high numbers and 20 that are not.

$EV = 1\left(\frac{18}{38}\right) + (-1)\left(\frac{20}{38}\right) = -\frac{2}{38} \approx -0.05263$, *or a loss of about $0.05 per $1 bet.*

B) We'll show two methods here.

a. Method 1 – With a $20 bet, the payout odds of 1:1 indicate we either win $20 or lose $20. Now use those values in the expected value equation:

$EV = 20\left(\frac{18}{38}\right) + (-20)\left(\frac{20}{38}\right) = -\frac{40}{38} \approx -1.05263$, *or a loss of about $1.05 per $20 bet.*

b. Method 2 – With a $20 bet, we'll find the EV for $1 bet and then multiply it by 20.

$20(EV) = 20\left(1\left(\frac{18}{38}\right) + (-1)\left(\frac{20}{38}\right)\right) = 20\left(-\frac{2}{38}\right) = -\frac{40}{38} \approx -1.05263$. Same result!

C) For this one, I'll use only method 1. The 2 number bet has payout of 17:1, so this means you will either win $17(35) = 595$ or lose 35. Now use those values in the expected value equation:

$EV = 595\left(\frac{2}{38}\right) + (-35)\left(\frac{36}{38}\right) = -\frac{70}{38} \approx -1.8421$, *or a loss of about $1.84 per $35 bet.*

D) For this one, I'll use only method 2. The 4-number bet pays out 8:1, so do the $1 bet expected value. $EV = 8\left(\frac{4}{38}\right) + (-1)\left(\frac{34}{38}\right) = -\frac{2}{38} \approx -0.05263$. Now multiply by the bet amount to get the final expected value: $EV = 57\left(-\frac{2}{38}\right) = -\frac{114}{38} = -3$, *or a loss of exactly $3 per $57 bet.*

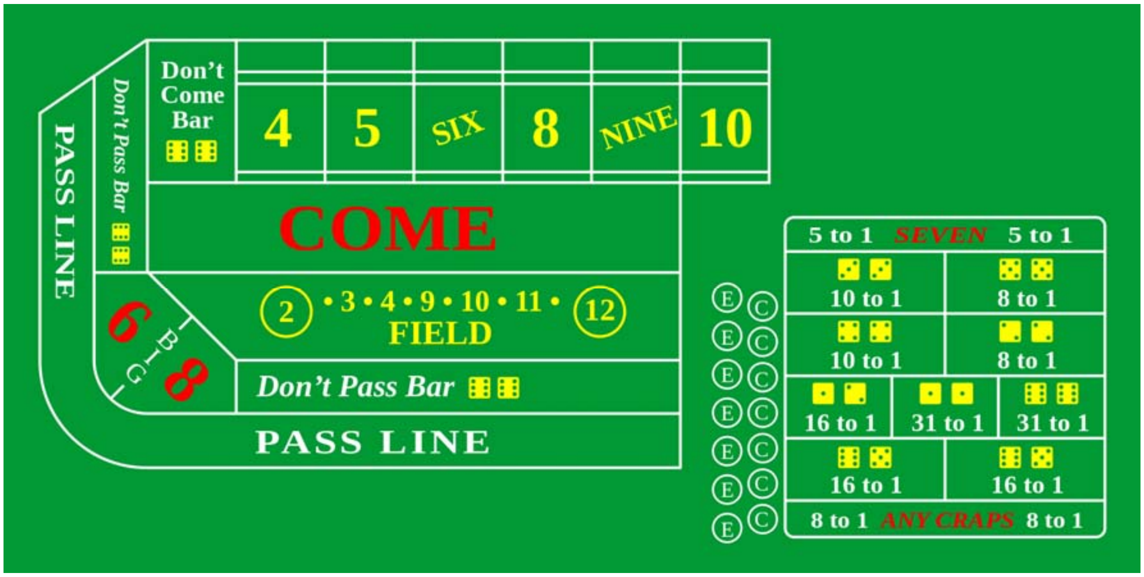
EXPLORE (3)! Find these Expected Values for Roulette. Use either method.

A) Bet $125 on Basket (5-number), payouts are 6:1.

B) Bet $133 on Double Street (6-number), payouts are 5:1.

C) Bet $683 on Street (3-number), payouts are 11:1.

Craps



Want to try it out: go to https://casino.bovada.lv/table-games and select "Craps" and play around!

Craps is a dice game where two 6-sided dice are rolled. There are many subtle nuances to the different bets, but the vast majority of bets are really bad (with decent payouts). For our class, we'll focus on one type of bet that has the highest expected value. The person rolling the dice is called the 'shooter.'

Craps Rules

- Two dice are rolled and the faces are added.
- Come-out roll (First roll) options:
    - If a 7 or 11 comes up, the shooter wins. ☺
    - If a 2, 3, or 12 comes up, then the shooter loses. ☹
    - If any other number is rolled (4, 5, 6, 8, 9, 10), that number is called the "Point."
- Point-roll options – these are done until the shooter wins or loses:
    - Now if a 7 comes up, the shooter loses.
    - If the point is rolled a second time, then the shooter wins!
    - Anything else is rolled, the dice go back to the shooter and we go again.

Main bets: Pass (Betting right) and Don't Pass (Betting wrong)

- If you bet on the Pass line, you are putting your money with the shooter. If the shooter wins, then you double your money. It's a 1:1 payout and you get the amount you bet back!
- If you bet on the Don't Pass line, you're putting your money against the shooter. If the shooter loses, then you double your money. It's a 1:1 payout. **NOTE:** For Don't Pass bets, the 12 that would cause the shooter to lose is actually a 'push.' You get your money back to bet again, but don't win anything.
    - Betting on Come and Don't Come have the same meaning as Pass/Don't Pass but are made after the Come-out roll. We won't focus on those bets at all.

While the two bets are very similar, the Don't Pass line is sometimes a tough social bet. The shooter wants to win, and can become frustrated when people bet against him/her. In a math class, we are more interested in the mathematics so we'll just try to find what bet is the best for us!

EXPLORE (4)! Now we'll find the Expected Value of the "Pass" bet in Craps.

A) Determine the probability of winning (7 or 11) on the first roll.

B) Determine the probability of losing (2, 3, or 12) on the first roll.

C) We will fill in the chart for the options for winning on the second roll.

Point	Number of ways to make the point	Probability	# of ways for 7 or point	Probability of point again	Probability of a win on second roll
4	3	$\frac{3}{36}$	$3+6=9$	$\frac{3}{9}$	$\frac{3}{36}\cdot\frac{3}{9}=\frac{9}{324}$
5	4	$\frac{4}{36}$	$4+6=10$	$\frac{4}{10}$	$\frac{4}{36}\cdot\frac{4}{10}=\frac{16}{360}$
6	5	$\frac{5}{36}$	$5+6=11$	$\frac{5}{11}$	$\frac{5}{36}\cdot\frac{5}{11}=\frac{25}{396}$
8	5	$\frac{5}{36}$	$5+6=11$	$\frac{5}{11}$	$\frac{5}{36}\cdot\frac{5}{11}=\frac{25}{396}$
9	4	$\frac{4}{36}$	$4+6=10$	$\frac{4}{10}$	$\frac{4}{36}\cdot\frac{4}{10}=\frac{16}{360}$
10	3	$\frac{3}{36}$	$3+6=9$	$\frac{3}{9}$	$\frac{3}{36}\cdot\frac{3}{9}=\frac{9}{324}$

D) Add up all the probabilities for winning on the first or second roll, the last column in part (C) and amount from part (A). Then use this to calculate the probability of losing.

E) Calculate the expected value for the Pass bet if your bet was $1 (remember it pays 1:1).

F) Calculate the expected value for the Pass bet if your bet was $57 (remember it pays 1:1).

G) Calculate the expected value for the Pass bet if your bet was $297 (remember it pays 1:1).

H) On the craps table, there are many other bets. Some are very basic like the bet on double-sixes. If you roll double sixes, then the payout is 31:1. Find the expected value of a $1 bet on double sixes.

Now that you have more experience with expected value, here's a few follow-up questions.

EXPLORE (5)!

A) Which game seems to have a better expected value between Craps and Roulette?

B) Can you use the $1 Craps bet expected value to determine the expected value for a $3 Craps bet? Explain how.

C) Why do we usually calculate the expected value for a $1 bet?

D) What is the expected value if you place a $500 bet in…
   a. Roulette
   b. Craps

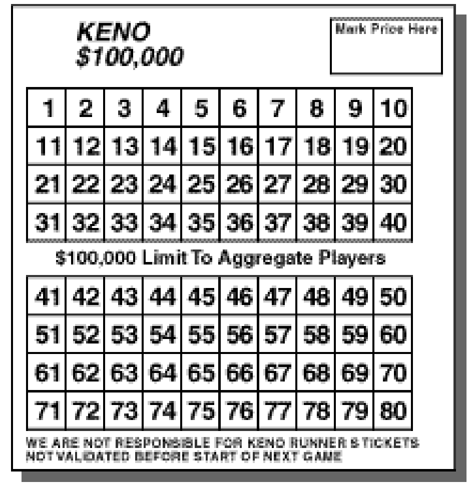
E) The percentage of your money that the house (casino) keeps is often known as the house edge, and is given as a percentage. What is the house edge for…
   a. ** Roulette

   b. Craps (Pass Line)

   c. Why does there need to be a house edge for the games?

   d. If you were playing roulette and the single number bet had payout odds of 37:1, what is the expected value and why is this a problem?

Keno



Keno's roots are in China, and the game was brought to the United States by Chinese immigrants in the late 1800s. It was originally played with 80 Chinese characters, which in the United States were replaced by 80 Arabic numerals. The game is pretty simple as each player chooses how many numbers they want to pick. These numbers are known as "spots." Examples include 3-spot, 4-spot, 5-spot, …, 10-spot. The more numbers you 'pick,' the higher the payouts. Once the player selects their numbers on a card (similar to the one shown here), then 20 winning numbers are randomly selected from a vat of 80 numbers.
[In Scott's last Vegas trip, some casinos had up to 15-spot with tremendous payouts!]

Years ago, the balls would be chosen like a Bingo game, but now nearly all Keno games are played with a computer randomly picking the numbers. Truly, this is like a lottery game with fixed jackpots.

- 80 total numbers
- 20 will be winning numbers
- 60 will be losing numbers
- The order of selection doesn't matter

In order to calculate the probability, let's review a probability question from the probability and counting section.

Example (2): In a group of 15 people, there are 5 women and 10 men applying for a job, but only 3 people will get it. What is the probability that the group selected has…

A) Exactly 3 women.
B) Exactly 1 woman.

Solution: First, use combinations to determine how many different ways a group of 3 people can be selected. Since there are 15 people and 3 to select, and since the job doesn't have any preferences between members, then we know that the order of selection doesn't matter. ${}_{15}C_3 = 455$, so there are 455 different ways to select the group of three people.

A) To select exactly 3 women, we need to think of how to choose the 3 women from the group of 5 women. Since there are 5 women and 3 to select, and since the order of selection doesn't matter, we can use combinations again. ${}_5C_3 = 10$, so there are 10 different ways to select the group of two women, meaning the probability is $\dfrac{{}_5C_3}{{}_{15}C_3} = \dfrac{10}{455} = \dfrac{2}{91} \approx 0.02198$.

B) In this last case, we need to select exactly 1 woman, which also means we need 2 men. Start by selecting the women: there are ${}_5C_1 = 5$ ways to select the 1 woman because there are 5 women to pick from. Now select the men: there are ${}_{10}C_2 = 45$ ways to select the 2 men. For each of the 45 ways to select the men, there are 5 ways to select the women, making a grand total of $({}_{10}C_2)({}_5C_1) = 45 \cdot 5 = 225$ ways to select exactly 1 woman and 2 men. The probability is computed as $\dfrac{({}_{10}C_2)({}_5C_1)}{{}_{15}C_3} = \dfrac{45 \cdot 5}{455} = \dfrac{225}{455} = \dfrac{45}{91} \approx 0.4945$.

Interactive Example (3): In a group of 80 people, there are 20 women and 60 men applying for a job, but only 5 people will get it. What is the probability that the group selected has…

A) Exactly 5 women. B) At least one man. C) Exactly 1 woman.

Solution: First, use combinations to determine how many different ways a group of 5 people can be selected. Since there are 80 people and 5 to select, and since the job doesn't have any preferences between members, then we know that the order of selection doesn't matter. ${}_{80}C_5 = 24,040,016$ so there are over 24 million different ways to select the group of five people.

A) Exactly 5 women means 5 women and 0 men. Put in the appropriate numbers and then compute the probability of exactly 5 women: $\dfrac{{}_{20}C_{\_\_} \times {}_{60}C_{\_\_}}{{}_{80}C_5} =$

B) To find at least one man, that would be the complement of having no men. Part (A) had no men selected. Use this to determine the probability of at least one man.

C) Exactly 1 woman means 1 woman and ____ men. Put in the appropriate numbers and then compute the probability of exactly 1 woman: $\dfrac{{}_{\_\_}C_{\_\_} \times {}_{\_\_}C_{\_\_}}{{}_{\_\_}C_{\_\_}} =$

Example (4): Play a 3-spot Keno game with standard Oregon prizes. The standard prizes are \$27 for matching three numbers and \$2 for matching two numbers. Since these are the amounts you walk away with, we can compute the expected value and then subtract our \$1 bet.

Solution: Let's break this into parts.

A) First, we'll find the probability of the \$27 prize. There are 3 winners and we select all of them while selecting 0 losers. $\dfrac{{}_{20}C_3 \times {}_{60}C_0}{{}_{80}C_3} = \dfrac{1,140}{82,160}$

B) Now we'll find the probability of the \$2 prize. There will be 2 winners and 1 loser, because in 3-spot keno, we always have to pick 3 numbers. $\dfrac{{}_{20}C_2 \times {}_{60}C_1}{{}_{80}C_3} = \dfrac{11,400}{82,160}$

C) Now to find the expected value, we multiply probability by value and add them up: don't forget to subtract the \$1 you bet; in Keno you don't get your \$1 back when you win!

Expected value: $(27)\left(\frac{1,140}{82,160}\right) + (2)\left(\frac{11,400}{82,160}\right) + (-1) \approx -0.34786$. So if you bet \$1 to play this game, then you would be losing about 35¢ every time you play on average. OUCH!

NOTE: Keno played live inside a casino often is better than state run games as they increase the big payouts to make them seem much better (although they often decrease the smaller payouts). Games on the Vegas strip have \$42 payout for matching 3 numbers and \$1 for matching 2 numbers. Expected value: $(42)\left(\frac{1,140}{82,160}\right) + (1)\left(\frac{11,400}{82,160}\right) + (-1) \approx -0.2785$.

Keno Payout Structures at different locations: Since California casinos are all different, we chose the state of Oregon's payouts, both standard (STD) or special (SPC) as well as Station casinos (in Vegas):

1-spot game ($1 bet)			
Match	OR-STD	OR-SPC	Vegas
1	$2.50	$2.50	$3.00

2-spot game ($1 bet)			
Match	OR-STD	OR-SPC	Vegas
2	$11	$11	$12

3-spot game ($1 bet)			
Match	OR-STD	OR-SPC	Vegas
3	$27	$47	$42
2	$2	$0	$1

4-spot game ($1 bet)			
Match	OR-STD	OR-SPC	Vegas
4	$72	$140	$130
3	$5	$5	$3
2	$1	$0	$1

5-spot game ($1 bet)			
Match	OR-STD	OR-SPC	Vegas
5	$465	$800	$700
4	$15	$12	$15
3	$2	$0	$1

6-spot game ($1 bet)			
Match	OR-STD	OR-SPC	Vegas
6	$1,600	$2,500	$2,000
5	$55	$90	$85
4	$5	$2	$2
3	$1	$0	$1

7-spot game ($1 bet)			
Match	OR-STD	OR-SPC	Vegas
7	$5,500	$7,500	$5,000
6	$150	$330	$300
5	$15	$20	$30
4	$2	$1	$2
3	$1	$0	$0

8-spot game ($1 bet)			
Match	OR-STD	OR-SPC	Vegas
8	$15,000	$25,000	$30,000
7	$600	$1,750	$1,500
6	$60	$75	$100
5	$10	$5	$5
4	$2	$0	$0

9-spot game ($1 bet)			
Match	OR-STD	OR-SPC	Vegas
9	$50,000	$75,000	$40,000
8	$3,000	$4,000	$4,000
7	$215	$300	$400
6	$25	$40	$30
5	$4	$2	$3
4	$1	$0	$0

10-spot game ($1 bet)			
Match	OR-STD	OR-SPC	Vegas
10	$200,000	$1,000,000	$1,000,000
9	$4,500	$22,000	$5,000
8	$500	$1,200	$1,000
7	$55	$140	$100
6	$10	$4	$10
5	$2	$0	$1
0	$5	$0	$0

EXPLORE (6)! Questions about Keno payouts.

A) Based on these payout amounts, which version looks like the most fun/exciting to play? Why?

B) Based on these payout amounts, which version looks like the most boring to play? Why?

C) Make a conjecture about which version has the best expected value and which has the worst.

Keno Expected Values at different locations: Since California casinos are all different, we chose the state of Oregon's payouts, both standard (STD) or special (SPC) as well as Station casinos (in Vegas). Blank values are here so the values can be computed in the homework. All values will be posted in the key.

1-spot game ($1 bet)		
OR-STD	OR-SPC	Vegas

2-spot game ($1 bet)		
OR-STD	OR-SPC	Vegas

3-spot game ($1 bet)		
OR-STD	OR-SPC	Vegas
-0.34786		

4-spot game ($1 bet)		
OR-STD	OR-SPC	Vegas

5-spot game ($1 bet)		
OR-STD	OR-SPC	Vegas
-0.35085	-0.33895	-0.28323

6-spot game ($1 bet)		
OR-STD	OR-SPC	Vegas
-0.35085	-0.34185	-0.29201

7-spot game ($1 bet)		
OR-STD	OR-SPC	Vegas
-0.34702	-0.35043	-0.29483

8-spot game ($1 bet)		
OR-STD	OR-SPC	Vegas
-0.35051	-0.34155	-0.30076

9-spot game ($1 bet)		
OR-STD	OR-SPC	Vegas
-0.35130	-0.34382	-0.33460

10-spot game ($1 bet)		
OR-STD	OR-SPC	Vegas
-0.34709	-0.31915	-0.39443

EXPLORE (7)! Use the table above to answer these questions.

A) Is it better to play the Vegas casino 10-spot game, or just take 50¢ out of your pocket and throw it on the ground every time you wanted to play? Which (over time) gives you more money? Explain.

B) Based on expected value, is it better to play the Vegas 3-spot Keno game repeatedly, or to just take a quarter out of your pocket every time you wanted to play and throw it on the ground?

EXPLORE (8)! Compare the three types of 3-spot Keno. Which is the best to play?

A) ** For the Oregon standard version, what is the expected value?

$(27)\left(\frac{1,140}{82,160}\right)+(2)\left(\frac{11,400}{82,160}\right)+(-1)\approx -0.34786$. So if you bet $1 to play this game, then you would be losing about 35¢ every time you play on average.

B) **(L)** For the Oregon special version, compute the expected value?

C) **(R)** For the Vegas casino version, compute the expected value?

Based on the expected values we've seen so far, it would make NO sense for an article to tell people to play this type of game, right? The New York Lottery plays Keno, but calls it "Quick-Draw," and the payouts are very similar to the ones shown in this Unit. In order to encourage more people to play, there was an advertisement that the payouts on the 4-spot game would be doubled on every Wednesday in November. An article in Math Horizons (Feb 2007) described this situation.
http://www2.stat.duke.edu/~kfl5/Lock2007.pdf

The original payouts were: 4 spots ($55), 3 spots ($5), and 2 spots ($1).

EXPLORE (9)! Answer the following questions about this situation.

A) ** One ticket allows you to play up to 20 games. If you played 20 games on a ticket and were charged $5 per game, how much would one ticket cost?

B) ** What is the expected value of a $5 bet on the standard 4-spot game?

C) What is the expected value of a $5 bet on every Wednesday in November?

D) Two math friends read about the ad and bought about 1,500 of the tickets listed above on each Wednesday in November. How much total did it cost them to play?

E) Using the expected value, what was their approximate profit for the month?

Lotteries

Lotteries are played in nearly every state, with profits given back to the state general fund and often a portion is given to education. In fact, MiraCosta receives proceeds from the lottery each year to offset our costs! But how do they work?

The major lotteries currently used by California are Powerball, MegaMillions, and Super Lotto Plus, but there are others. Fortunately, we don't need to think about an entirely new system as both of these lotteries are basically Keno!

Lottery Name	Number of 1^{st} type	Number you select	Number of 2^{nd} type	Number you select	Cost to Play	Jackpot Starts at
Powerball (current)	69	5	26	1	$2	$40 M + $10 M*
Powerball (before October 2015)	59	5	35	1	$2	$40 M + $10 M*
MegaMillions	75	5	15	1	$1	$15 M + $5 M*
Super Lotto Plus	47	5	27	1	$1	$7 M + $1 M

NOTE: These are progressive Lotteries which means that the prize will go up each week that there is not a winner. Those marked with a * are the minimum increase between each drawing, but based on sales, that amount could increase. The most famous of these happened between November 2015 and January 2016, when no one was the winner of Powerball for 19 drawings in a row. While $10 million is the guaranteed increase between the drawings, the increase was actually $45 million (12/30/15), then $34 million (1/2/16), then $166 million (1/6/16), followed by $449.8 million (1/9/16), and finally $550.2 million (1/13/16) when the jackpot was won by 3 people in different states. The stated prize was $1.5 billion and was split 3 ways.

On October 7, 2015, the states participating in the Powerball enacted changes to the way it was played to increase the number of people playing the game. The payout structure was also changed to actually increase the odds of winning something. As we can see from the result just a few months later, their goal was met; prior to the $1.5 billion prize, the largest Powerball win was $590.5 million in May of 2013.

EXPLORE (10)! Did you play the Powerball during this big rush? Why or why not?

California is the only state participating in Powerball that doesn't fix the prize amounts. Other states pay out fixed amounts for 2^{nd}, 3^{rd}, 4^{th}, etc. $1,000,000 for second place is paid every time in all states but California, where we base the payout on the number of winners/ticket sales – sometimes more, sometimes less.

- December 23, 2015 – $1,363,153.
- January 13, 2016 – $638,146.
- January 16, 2016 – $231,203.
- February 6, 2016 – $1,223,935.

Yup. California is just that crazy! Because of this added complexity, when we compute the expected value for these games, we will use fixed dollar amounts as prizes and pretend we are in another state. Ha!

Here's the standard (non-California) payout structure for Powerball:

White Matches	Powerball	Prize
5	1	Grand Prize (Varies)
5	0	$1,000,000
4	1	$50,000
4	0	$100
3	1	$100
3	0	$7
2	1	$7
1	1	$4
0	1	$4

EXPLORE (11)! Now use this to help us compute the probability of winning and set up expected values.

Prize	White Matches	Powerball	Probability (as many decimals as possible)
Grand Prize ($$ Varies)	5	1	$\frac{{}_5C_5 \times {}_{64}C_0}{{}_{69}C_5} \cdot \frac{{}_1C_1 \times {}_{25}C_0}{{}_{26}C_1} \approx 0.000\ 000\ 003\ 422$
$1,000,000	5	0	$\frac{{}_5C_5 \times {}_{64}C_0}{{}_{69}C_5} \cdot \frac{{}_1C_0 \times {}_{25}C_1}{{}_{26}C_1} \approx 0.000\ 000\ 085\ 574$
$50,000	4	1	$\frac{{}_5C_4 \times {}_{64}C_1}{{}_{69}C_5} \cdot \frac{{}_1C_1 \times {}_{25}C_0}{{}_{26}C_1} \approx 0.000\ 001\ 095\ 135$
$100	4	0	$\frac{{}_5C_4 \times {}_{64}C_1}{{}_{69}C_5} \cdot \frac{{}_1C_0 \times {}_{25}C_1}{{}_{26}C_1} \approx 0.000\ 027\ 378\ 383$
$100	3	1	$\frac{{}_5C_3 \times {}_{64}C_2}{{}_{69}C_5} \cdot \frac{{}_1C_1 \times {}_{25}C_0}{{}_{26}C_1} \approx 0.000\ 0689\ 935\ 239$
$7	3	0	$\frac{{}_5C_3 \times {}_{64}C_2}{{}_{69}C_5} \cdot \frac{{}_1C_0 \times {}_{25}C_1}{{}_{26}C_1} \approx 0.001\ 724\ 838\ 098$
$7	2	1	$\frac{{}_5C_2 \times {}_{64}C_3}{{}_{69}C_5} \cdot \frac{{}_1C_1 \times {}_{25}C_0}{{}_{26}C_1} \approx 0.001\ 425\ 866\ 161$
$4	1	1	
$4	0	1	

EXPLORE (12)! Using the table, what is the probability of …

A) Winning the Grand Prize or $1,000,000?

C) Winning less than $10?

B) Winning $50,000 or more?

D) Winning anything?

E) Once you find the value in (D), press the x^{-1} button on the calculator. What value shows up on screen?

F) Use the number you just found to create a probability as 1 out of _________________.

G) From the previous exploration, you found the probability to be 1 out of 24.867 that you will win something. Compare this to the listed **odds** of winning, which a Powerball ticket lists as 1 in 24.87. Does the math seem to check out?

H) Is the Powerball ticket correct to claim the odds are 1 in 24.87? Explain.

I) How much would it cost to "cover the numbers" – which is when you buy a ticket representing every number combination possible?

   a. If it took 3 seconds to buy one number combination, and you stayed at a machine running number combinations from Wednesday night at 10pm until Saturday at 6pm, how many ticket combinations could you buy?

   b. How many machines would you need to be running to purchase all of the number combinations?

EXPLORE (13)! Use the spreadsheet "Powerball and Martingales" and the "Powerball" tab to compute an expected value for Powerball. The grand prize for February 6, 2016 was $136,000,000.

A) What was the expected value for the $136,000,000 Powerball drawing? List all.

B) If the jackpot rose to $300 million, find the new expected value. List all.

C) If the jackpot rose to $1.5 billion, find the new expected value. List all.

D) What is the expected value of the standard jackpot of $40,000,000? List all.

E) Since these are all expected values on $2 bets, what is the expected value per dollar? This will allow us to compare it to the other games in this unit.

a.

b.

c.

d.

F) Now on the sheet, change from 1 to 2 in the "Type of Powerball." Describe what you discover:

a. Is the expected value higher or lower with the new Powerball game?

b. Is the probability of winning higher or lower with the new Powerball game?

c. Based on your answers, explain why the Powerball leaders wanted to make the change?

<u>EXPLORE (14)!</u> Wrap up on Lotteries/Keno/Roulette/Craps.

A) Do you feel it is a good idea to play the lottery, based on the expected value?

B) Based on the expected value of Powerball, would you rather play Keno or the Powerball? Why?

C) If the expected value of a $1 lottery is $ – 0.85 and the expected value of a $2 Powerball ticket is calculated to be $ – 1.48, which option is the better choice? Explain.

D) What is the message you've learned in this class about state-run lotteries, like Powerball?

E) If the money from Powerball is sent to the state and a large portion then sent to educate students, and students learn the information you are learning now, what do you see as a result? Explain the irony.

F) Out of the games listed (Roulette, Craps, Keno, Lottery), if you wanted to play the games with $200 for the longest time, which game would you choose and why?

G) Is there some entertainment value in gambling? What about in going to a movie?If you paid $15 to go to a movie, how much money will you walk away with?

H) Knowing what you now know, will you ever go out 'gambling' or not? Explain your answer.

3.12: Fun Probability Applications

Now that you've seen how probability can apply to many aspects of life, and how it relates to your future in statistics. So it's time to have a bit of fun with some probability questions that may not always apply to anything in your life but will force you to think critically about the concepts we learned so far.

EXPLORE (1)! Application #1 – Nerf Dart Guns

My kids have a Nerf gun that has a rotating cylinder (image to the right). In order to see who will do the chores for the week, the kids load exactly one Nerf dart in the cylinder and then spin it. They aim the Nerf gun at the chore card and pull the trigger. If the gun fires, they get to do that chore! There are 6 cylinders in the gun…

A) If we load one dart, what is the probability that the dart is fired?

B) If the gun didn't go off, is it better to pull the trigger again, or spin the cylinder and then pull the trigger?

C) In order to make this game go quicker, sometimes I will put in 2 darts, right next to each other.
   a. For this situation, what is the probability that the dart is fired?

   b. If the gun doesn't go off, then the next daughter is up. She is now holding the Nerf gun which didn't go off on the last trigger pull – is she better off to pull the trigger without spinning the cylinder, or is it better for her to spin the cylinder and then pull the trigger?

D) Does this seem strange? Explain.

EXPLORE (2)! Application #2 - Blackjack

There was a game with letters in a bag. The letters put in were S-O-S and the contestant had to draw the S first, then O second, then S third.

A) With two "S" and one "O" in the bag, what is the probability of winning the game (without replacement)?

B) With two "S" and one "O" in the bag, what is the probability of winning the game (with replacement)?

C) **(R)** If the host doubled the number of each, so there were four "S" and two "O" in the bag, what is the probability of winning the game (without replacement)?

D) **(L)** If the host doubled the number of each, so there were four "S" and two "O" in the bag, what is the probability of winning the game (with replacement)?

E) **(L)** If there were 200 "S" and 100 "O" letters in the bag, what is the probability of winning the game (without replacement)?

F) **(R)** If there were 200 "S" and 100 "O" letters in the bag, what is the probability of winning the game (with replacement)?

G) Based on this example, does increasing the number of options while keeping the same ratio actually help your chances or hurt them?

Let's play again, but this time… BLACKJACK!

A) There are four "Aces" worth 11 and sixteen cards worth 10 in a standard deck. Determine the probability of drawing two cards and getting Blackjack (total of 21).

B) If we put 6 decks of cards together, determine the new probability of drawing two cards and getting Blackjack (total of 21).

C) Why do casinos put multiple decks together when dealing Blackjack?

EXPLORE (3)! Application #3 – Gambler's Ruin

A friend of mine came up with a nice strategy that he said would beat the casino in a game of roulette or craps. He would bet the table minimum ($5) on something that was close to an even bet, so the payout odds would be 1:1. If he won, then he would pocket the winnings and start again. If he didn't win, then he would double his bet to $10 and repeat. Complete the table to track the possibilities for his next bets. Remember that a table with a minimum bet of $5 often has a maximum bet of $200. Instead of trying to play forever, he picked a dollar amount like $50 and then once he hit that amount, he walked away!

EXPLORE (3)! Fill out this chart to see why the friend is so happy with this system.

Next bet	Overall status (win)	Overall status (loss)
$5	$5	– $5
$10	$5	– $15
$20		
$40		
$80		

Use the spreadsheet "Powerball and Martingales" and the "Martingale - Roulette" tab for the rest of this page. Once you read the directions, set up the computer and play the game.

A) Let's start into the casino with $500. Did you win or lose? (look to the purple box with white type)

B) From this class, determine the probability that you will lose 6 times in a row?

C) Determine the probability that you will win at least once in 6 spins of the wheel?

D) How did the class do – class results will be put on the board. Did the *casino* win or lose? Did *you* win or lose?

3.13: Expected Value and Gambling Wrap-up.

Gambling can be fun and it can also be costly. We hope that you are able to now understand the way that many games work so that you can make an informed decision about whether or not you would like to gamble.

In writing about this, we felt compelled to describe some of the manipulations that casinos use to keep you spending money. Many of these are psychological in nature:

- Depending on how much you're spending, casinos will give you free food and free drinks to keep going. Free alcoholic drinks are often given to reduce your inhibitions and decrease your decision making ability.
- Most casinos eliminate natural light outside of the lobby and have removed clocks from the walls. The idea is to create an environment where you can't gauge how long you've been playing. All of your natural cues about the sun going down (or coming up) are gone.
- Machines are set to make loud noises to simulate winning a prize, with flashing lights grabbing your attention. This is intended to feed on the gambler's fallacy that "everyone else is winning and I must be due to win too!"
- Casinos pander to superstition as well. Many casinos will show the last 10 spins of the roulette wheel as a way to indicate 'trends' and make it seem easier to win.

But they do this for a reason. Take Roulette for example, with an expected value to the casino of 5.3 cents per $1 bet.

EXPLORE (1)! Analyze these situations.

A) If there is $300,000 bet on a roulette wheel in one day, how much profit will the casino expect?

B) Using the information above, but now the casino has 10 roulette wheels, how much profit will the casino expect?

C) Which seems to take more of your money – a state-run lottery or a casino? Explain your answer related to expected value.

D) Is there a casino system of betting that will 'beat' the house?

EXPLORE (2)! Analyze these situations.

A) How much money would you expect to have if you invested $50 per week for 20 years into an account paying 5.35% annual interest compounded weekly?

B) How much money would you expect to have if you invested $50 per week into Powerball tickets with an expected value of $ – 1.15 per $2 bet, and you did this for 20 years?

C) How much money would you expect to have if you put $50 per week for 20 years into a safe deposit box?

D) If you took all the money from the safe deposit box (in the previous part) after 20 years and used all of it to buy Powerball tickets, what is your chance of winning one of the major prizes (Grand Prize, $1,000,000, or $50,000)?

- Jackpot: probability is 1 in 292,201,338
- $1,000,000: probability is about 1 in 11,688,054
- $50,000: probability is about 1 in 913,129

E) Which of the previous options will leave you with the most money? Explain.

F) Which of the previous options will leave you with the least money? Explain

G) Which option has the best chance of making you rich? Explain.

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